Monetary Policy in Oil-Producing Economies

by

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Abstract

Most oil-producing economies have a strong dependence on oil revenues for their economic performance and stability. This paper develops a general equilibrium model of an oil-producing economy that takes into account a new transmission channel for oil price shocks. This transmission channel can be described as an income effect generated by oil revenues, and the model shows that its role is important to fully understand monetary policy in these economies. I first present a static model that illustrates that a tension is present in such economies when faced with increases in oil prices. This tension arises, on the one hand, from the contractionary effects of higher oil prices and, on the other hand, from the income effect generated by the increased oil revenues. I then present and solve a dynamic model with price rigidities in a two-sector economy with an oil sector. I find that the Phillips curve includes a measure of oil income that is responsible for additional inflationary pressures. Impulse response functions show that, in terms of consumption and inflation stabilization, the economy responds better to a Taylor rule that reacts to both final goods production and oil production than to a Taylor rule that reacts to final goods production only, although in the former the volatility implied for non-oil output is higher. I also explore welfare-based optimal monetary policy in this framework and conclude that a central bank can stabilize both inflation and output without trade-off by reacting optimally to inflation and the output

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gap. Additionally, among Taylor-type rules, a rule that reacts to consumption and not only to final goods production is welfare superior.

1 Introduction

Recent economic and geopolitical events have caused significant impact on oil prices, with US crude reaching record highs above $78 dollars per barrel on July 14, 2006. Although the consequences and transmission mechanisms of oil price shocks have been investigated to some extent for developed economies, the effects of oil price shocks on oil-producing emerging market economies (EMEs) have yet to be explored.

Oil price shocks, defined as unexpected changes in the price of oil, have additional effects on oil-producing EMEs compared to their impact on developed economies. The economic implications of an increase in the price of oil in a developed economy are, mainly, the creation of inflationary pressures due to increases in production costs, a decrease in productivity levels, and a contractionary effect on output due to a reduction in aggregate demand. In contrast, in an oil-producing EME, an oil price shock carries an additional effect. Specifically, the economy as a whole receives a positive wealth effect through better terms of trade and an increase in oil revenues.

The contractionary and cost-push effects have been studied for the case of developed economies, but the wealth effect has not. Potentially, because of the nature of this second channel of transmission, additional inflationary pressures may be present due to the effects on marginal cost and aggregate demand. Naturally, the question of whether there is a positive or negative effect on output also arises. This line of reasoning implies that the role of monetary policy in these economies may be different compared to the cases when only the standard channels of shock transmission are considered.

This paper develops a general equilibrium model for oil-producing EMEs that takes into account the additional wealth effect generated by an oil price shock. I first construct a static model that explicitly models an oil production sector. The oil sector sells part of its production to the final goods producing firms and the remaining oil production is sold abroad. This formulation allows for an illustration of the effects of an oil price shock on the relevant economic variables and shows that, depending on the elasticity of labor supply, the effect of an increase in the price of oil can have either a contractionary or an expansionary effect on output. This static model will aid in the understanding and interpretation of the dynamic general equilibrium model.

The dynamic model consists of a final goods sector characterized by monopolistic competition in a price-setting environment in which firms adjust prices as in Calvo (1983). Additionally, there is an oil sector operating a decreasing returns to scale technology and selling its output in competitive world markets. I derive the New Keynesian Phillips curve for this framework and illustrate that additional inflationary pressures exist in this economy. In particular, the Phillips curve includes a term that is a measure of oil income that exerts extra pressures on inflation. This derivation also confirms that the standard cost push effects of an oil price shock are present in this economy. In the model, depending on the monetary policy regime, the effect of an oil price shock on output and consumption can be considerably different. In particular, in terms of consumption and inflation stabilization, the economy responds better to a Taylor rule that reacts to consumption than to a Taylor rule that reacts to final goods production only. The trade-off is that the volatility implied for non-oil output is higher in the former. Hence, the effects on inflation, final goods production and consumption are highly sensitive, and can be considerably different in magnitude, depending on the type of monetary policy rule specified. I also calculate impulse responses to productivity shocks and conclude, as expected, that an increase in productivity in the oil sector has qualitatively similar effects as those caused by an oil price increase.

I then turn to the study of optimal monetary policy. Specifically, the model allows the study of welfare-based optimal monetary policy. To do this, I derive a loss function by writing the household's utility function as a second order Taylor expansion. I conclude that a central bank can stabilize both inflation and output without trade-off by reacting optimally to inflation and the output gap. Additionally, among Taylor-type rules, a rule that reacts to consumption and not only to final goods production is welfare superior.

The remainder of the paper is organized as follows. Section 2 presents some facts about oil-producing EMEs as well as the relevant literature review. Section 3 explores a static version of the model that will serve to illustrate some main results and will aid in building intuition for the dynamic model. Section 4 presents the dynamic model and derives some theoretical results. Section 5 includes the calibration of the model for a representative economy and section 6 concludes. Finally, the appendices contain impulse response plots as well as the proofs of the propositions and other technical derivations.

2 Some facts about oil-producing EMEs

Of the top fourteen world oil producers of 2004, only four of them (United States, Canada, Norway and United Kingdom) are considered developed economies as cataloged by the World Bank, the International Monetary Fund, and the Central Intelligence Agency. Even high income countries such as Saudi Arabia are not considered developed economies because of their tight dependence on oil revenues. Table 1 illustrates this point. It shows the top world oil net exporters of 2004. The shares of net oil exports and oil production on GDP depict the fact that these economies rely on the performance of their oil sector for economic stability.

Furthermore, compared to the shares shown in Table 1, the shares of oil revenues on fiscal revenues tend to be even greater in these economies. As specific examples of this fiscal dependence, according to the Energy Information Administration, between 75%-80% of Venezuela's exports rely on oil and contribute up to 45%-50% of government revenues. For Mexico, the average contribution of oil revenues on fiscal revenues for the period of 1998-2004 was approximately 30%. Therefore, oil prices play a critical role in the stability of government accounts in these economies and so condition fiscal policy, which in turn conditions monetary policy.

In general, emerging market economies and developing economies are more vulnerable to increases in oil prices than developed nations. According to a 2004 analysis by the

Country	Net Oil Exports[1]	Net Oil Exports (% of GDP)	Oil production (% of GDP)
Saudi Arabia	8.73	43.1	52.3
Russia	6.67	14.3	19.9
Norway	2.91	16.2	17.7
Iran	2.55	21.0	33.5
Venezuela	2.36	29.3	36.9
United Arab Emirates	2.33	27.4	31.2
Kuwait	2.2	47.0	53.0
Nigeria	2.19	46.6	53.2
Mexico	1.8	3.0	6.5
Algeria	1.68	30.7	35.6
Iraq	1.48	53.0	72.0
Libya	1.34	59.0	70.8
Kazakhstan	1.06	32.5	39.8
Qatar	1.02	47.0	45.6

(1) Million barrels per day.

Source: British Petroleum, Economist Intelligence Unit, CIA, EIA, and own calculations.

Table 1: Top World Oil Net Exporters, 2004

International Energy Agency (IEA) based on IMF estimations, for a sample of developing economies, a \$10 permanent increase in the price per barrel of oil would imply a 1.5% reduction in GDP. The analogous effect on OECD countries amounts to only a .3%-.4% GDP reduction. Similarly, the cost push pressures of a \$10 increase in oil prices would imply an increase in inflation of 1%-2% for the non-OECD sample and a .5% increase for the OECD countries. Moreover, the IEA study mentions three facts that explain why the impact of higher oil prices on emerging market economies is more severe than on developed nations. First, non-OECD countries are not able to switch rapidly and efficiently to other forms of energy. Second, these countries tend to be more energy intensive and less efficient in their energy usage. And third, non-OECD countries require twice as much energy input for a unit of output than do OECD countries.

Oil price changes also affect aggregate demand in an EME. Tijerina-Guajardo and Pagán (2003) have pointed out that the Mexican government has reduced government spending aggressively after facing significant oil price reductions in order to avoid a threatening deficit.

They notice that a 16.9% decrease in the price of Mexican oil between January and February 1998 led to a decrease in government spending of 0.5% of GDP. Their study also emphasizes that, for Mexico, GDP significantly responds to a shock in oil duties.

The correlation between increases in oil prices and contractions in the GDP has been extensively documented[1] for the US and other developed economies. However, Hamilton and Herrera (2004) have pointed out that there is no general consensus as to what this correlation means. Nevertheless, for a group of developed economies that includes the US, Bjørnland (2000) finds that oil price shocks have a significant negative effect on output.

Turning our attention to the relevant literature concerning monetary policy in oil-producing economies, DeLong (1997), Barsky and Kilian (1999), Hooker (1999), and Clarida, Galí and Gertler (2000), are among those who suggest that there is a role for monetary policy when dealing with oil price shocks. Hunt, Isard and Laxton (2001) use the IMF multicountry model to suggest that the effects of oil price shocks on economic activity can be limited if appropriate monetary rules are chosen.

There has been considerable debate as to whether monetary policy, and not oil price shocks per se, has been responsible for US recessions that have followed significant oil price increases. Among those who have attributed the source of these recessions to the Federal Reserve's response are Bohi (1989) and Bernanke, Gertler and Watson (1997). In contrast, Hamilton and Herrera (2001, 2004), among others, argue that oil price shocks were the source of those recessions, emphasizing that the potential of monetary policy to counteract these shocks is minimal. Aguiar-Conraria and Wen (2005) specify that in order to explain the sharp US recessions of 1974-75 and 1976-78 with oil price movements, it is necessary to take into account a multiplier-accelerator mechanism that arises in a model characterized by monopolistic competition with increasing returns to scale. Rotemberg and Woodford (1996) show that a model with imperfect competition can explain the effects of oil price shocks

[1] Relevant research in this direction includes work by Rasche and Tatom (1981), Hamilton (1983, 1985, 1996, 2003), Burbidge and Harrison (1984), Santini (1985), Gisser and Goodwin (1986), Loungani (1986), Tatom (1988) and Mork (1989), among others. For an extensive list of references studying this correlation, see Hamilton and Herrera (2004).

on output and real wages much better than a model that considers perfectly competitive markets. In contrast, Finn (2000) argues that perfect competition is enough to explain these effects.

There have been a few recent models that analyze oil shocks in a New Keynesian framework. Perhaps one of the first attempts in modeling the effects of an oil shock in a dynamic general equilibrium setting that allows an explicit role for monetary policy is the work by Kamps and Pierdzioch (2002). Their model studies the interaction between oil price shocks and monetary policy in a small open economy framework. In particular, they study the relative performance of alternative monetary policy rules and suggest core inflation targeting as the appropriate rule for an economy that faces an oil shock. Chang (2005) explores monetary policy towards oil shocks in a New Keynesian closed economy setting. She concludes that the optimal monetary policy rule is sensitive to the structure of the economy and, in general, contractionary in response to significant oil price increases.

Acknowledging the existing literature discussed above, there is no common framework or general consensus that addresses the role of oil shocks and monetary policy in a New Keynesian model for developed economies. Moreover, there appears to be little or no research focusing on the study of oil price shocks in oil-producing, oil-dependent economies. This paper presents an attempt to model the effects of oil price shocks on oil-producing economies that depend, to an important degree, on oil revenues for their economic performance and stability. The model that I develop contributes to this literature by considering a wealth effect that is present in oil-producing EMEs, and that acts as an additional channel of transmission for oil price shocks.

3 Static Model

I start with a simple static model that explains the different outcomes that changes in oil prices can induce in an oil-producing economy. The economy is populated by a representative household, a final good producing firm, and an oil-producing firm. The household consumes the final good and provides labor to both the final good sector and the oil sector. Labor is perfectly mobile between the oil and non-oil sectors, and workers in both sectors are subject to the same competitive wage. The household owns both firms and receives profits from them. The final good producing firm buys oil from the oil sector at a world-determined, exogenously given price. The oil sector maximizes profits, satisfies domestic demand for oil, and sells the residual to the rest of the world. No distortions are present in this model and all markets are competitive.

Three key features distinguish this setup from an otherwise standard model. First, the oil sector competes for labor against the final good sector. Second, there is an international oil market where the real price of oil is determined by cartel negotiations or other events that are exogenous to the economy. Specifically, OPEC, as well as economic and political events around the world, are responsible for the major changes in oil prices. Third, changes in oil prices produce a wealth effect in oil-producing economies. These particular characteristics of the static model will allow us to study the effects of oil price shocks on oil-producing economies. The following sections describe the model in greater detail.

3.1 Consumers

The economy is inhabited by a representative household that derives utility from consumption (C_t) and disutility from labor[2] (N_t^S). The representative household seeks to maximize

$$U_t = u(C_t, N_t^S) \tag{1}$$

$$\text{s.t. : } S_t \geq 0, \tag{2}$$

$$S_t \equiv W_t N_t^S + \Pi_t + \Pi_t^O - P_t C_t. \tag{3}$$

The function $u(\cdot)$ is concave and strictly increasing in C_t, and is a decreasing, concave function of N_t^S. The household receives labor income $W_t N_t^S$, where W_t denotes the nominal

[2] $N_t^S = N_t^{supply} + N_t^{O\ supply}$, where N_t^{supply} is labor supplied to the final good sector and $N_t^{O\ supply}$ is labor supplied to the oil sector.

wage, and also receives profits Π_t and Π_t^O from both the final good producing firm and the oil-producing firm, respectively. It uses these funds to finance consumption C_t, which costs P_t per unit. S_t denotes savings, which in the static economy have to be non-negative.

The first order conditions for the household imply:

$$\frac{u_N(C_t, N_t^S)}{u_C(C_t, N_t^S)} = \frac{W_t}{P_t}, \tag{4}$$

where first derivatives of $u(C_t, N_t^S)$ with respect to consumption and labor are denoted by $u_C(C_t, N_t^S)$ and $u_N(C_t, N_t^S)$, respectively. Equation (4), and the condition $S_t = 0$, characterize optimal consumption and labor supply decisions for the household.

3.2 Firms

A final good Y_t is produced by a perfectly competitive representative firm that uses oil O_t and labor N_t as inputs. The firm buys oil in the competitive market at price P_t^O. It operates a constant returns to scale Cobb-Douglas production function:

$$Y_t = A_t N_t^{\alpha} O_t^{1-\alpha}, \tag{5}$$

where $\alpha \epsilon (0, 1)$ and A_t denotes total factor productivity. The firm's objective is to maximize profits taking P_t, W_t and P_t^O as given:

$$\max_{N_t, O_t} \left[P_t Y_t - W_t N_t - P_t^O O_t \right]. \tag{6}$$

By defining labor-intensive oil as $o_t \equiv \frac{O_t}{N_t}$, the firm's first order conditions can be written as

$$P_t A_t o_t^{1-\alpha} - W_t - P_t^O o_t = 0, \tag{7}$$

$$(1-\alpha) P_t A_t o_t^{-\alpha} - P_t^O = 0. \tag{8}$$

3.3 Oil sector

The oil-producing firm maximizes profits by taking prices P_t^O and W_t as given, and determines total oil supply O_t^S in the economy. It operates a decreasing returns to scale technology. P_t^O is determined by world markets and cartel behavior and is therefore assumed to be exogenous. The oil firm satisfies oil demand of the final good producing firm and sells the remaining oil to the rest of the world. The problem faced by the oil sector can therefore be summarized as:

$$\max_{N_t^O} \left(P_t^O O_t^S - W_t N_t^O \right) \tag{9}$$

$$s.t.: O_t^S = Z_t \left(N_t^O \right)^{\nu}, \tag{10}$$

where $\nu \in (0,1)$, Z_t measures the productivity level of the oil firm and N_t^O denotes oil sector labor. The first order condition implies

$$\nu Z_t P_t^O \left(N_t^O \right)^{\nu - 1} = W_t, \tag{11}$$

as well as the profit function

$$\Pi_t^O = \nu^{\frac{\nu}{1-\nu}} (1-\nu) \frac{\left(Z_t P_t^O \right)^{\frac{1}{1-\nu}}}{W_t^{\frac{\nu}{1-\nu}}}. \tag{12}$$

Equation (11) determines labor demand for the oil sector as a function of relative prices. The assumption of decreasing returns to scale implied by (10) plays an important role in the model. It implies a unique maximum for the oil sector's maximization problem and guarantees the existence of positive profits. The latter assertion can easily be noted from (12). These profits are allocated to the household and used to purchase the final consumption good. Note that, if one were to assume a constant returns to scale technology (i.e. $\nu = 1$), (11) would imply that $Z_t P_t^O = W_t$. In other words, real wages would be determined only by the exogenous value of P_t^O (for a given value of Z_t). This would be both an unrealistic and undesirable property of the model given that, in principle, the equilibrium real wage should be determined by market clearing conditions in the labor market.

3.4 Market clearing conditions and resource constraint

The economy's resource constraint is given by

$$C_t = Y_t + \Omega_t, \tag{13}$$

where

$$\Omega_t \equiv \frac{P_t^O O_t^*}{P_t} \tag{14}$$

and O_t^* denotes oil exports:

$$O_t^* \equiv O_t^S - O_t. \tag{15}$$

Ω_t can be interpreted as a measure of period t wealth derived from oil. It is the consumption-good equivalent of the income obtained from the sales of oil abroad. Equation (13) states that total consumption equals total production in the economy plus a lump sum of goods obtained from converting oil export revenues into their consumption-good equivalent. By rewriting (13) as

$$P_t C_t = P_t Y_t + P_t^O O_t^*, \tag{16}$$

the resource constraint can similarly be interpreted as follows: the left-hand side is the total nominal value of consumption, while the right-hand side is the sum of the nominal value of final good production and the total revenue obtained for oil exports. Equivalently, one can also think of this economy as existing in an environment in which oil exports are used to purchase imports from abroad. In this case, we can notice that two assumptions are implicit in (13) and (14). One of these assumptions is that trade is balanced. The other assumption is that purchasing power parity (PPP) holds. In order to see this, recall that

P_t^O is denominated in domestic currency and note that, by defining M_t as imports in the oil-producing economy and P_t^M as the price, denominated in foreign currency, of one unit of imports, we can write $P_t^M M_t = P_t^O O_t^* / \varepsilon_t$, where ε_t, the nominal exchange rate, is the price of a unit of foreign currency in terms of domestic currency. Then we have $M_t = \frac{P_t^O O_t^*}{\varepsilon_t P_t^M}$, which equals $\frac{P_t^O O_t^*}{P_t}$ under PPP and is identical to (14)[3].

For the labor market, total labor supplied by the household must clear the labor demands in both the final good sector and the oil sector

$$N_t^S = N_t + N_t^O \tag{17}$$

This finalizes the setup for the static model. In what follows, I make the assumption that the real price of oil is exogenous to the economy.

3.5 Competitive equilibrium

In general equilibrium, only real prices can be determined. Therefore, taking C_t as the numeraire of this economy, we can assume, without loss of generality, that $P_t = 1$. The solution of this economy is defined as follows:

Definition 1 *A Walrasian equilibrium is an allocation vector* $[C_t, W_t, Y_t, N_t^S, N_t, N_t^O, O_t^S, O_t, O_t^*]$ *such that (a) given real prices* W_t *and* P_t^o*, the allocation solves the optimum problems of the household ((2) and (4)), the final good producing firm ((7) and (8)), and the oil-producing firm ((10) and (11)) and, (b) market clearing conditions (13), (15) and (17) are satisfied.*

This is a system of 9 equations in 9 unknowns. We notice that since household preferences are locally nonsatiated, the first fundamental theorem of welfare economics tells us that the competitive equilibrium defined above is Pareto-efficient. I now proceed to describe some general properties of the model.

[3] Alternatively, one can think of this economy as having a linear technology that transforms exports into imports.

3.6 Solution and some important results

This section presents the main results obtained from the static model. As a first point, given that P_t^O and A_t are exogenous, the equilibrium levels of real wage and labor-intensive oil of this economy are determined only by the final goods firm. To see this, note that (7) and (8) form a system of two equations in two unknowns $(o_{t,}W_t)$. Solving for W_t and o_t yields

$$W_t = \alpha A_t^{1/\alpha} \left[\frac{(1-\alpha)}{P_t^O} \right]^{\frac{1-\alpha}{\alpha}}, \tag{18}$$

$$o_t = \left[\frac{A_t(1-\alpha)}{P_t^O} \right]^{\frac{1}{\alpha}}. \tag{19}$$

Given that the firm is competitive, real marginal cost is equal to one. Therefore, since P_t^O is exogenous, in order for the firm to exist under competitive markets, W_t is forced to make the real marginal cost equal to one. Equation (18) is the mathematical equivalent of this statement. Note that this result shows that the real wage is decreasing in the real price of oil. Likewise, equation (19) states that the equilibrium level of labor-intensive oil is negatively related to the real price of oil. This is a direct consequence of the fact that when oil prices increase, oil is relatively more expensive than labor and therefore factor substitution will take place by increasing labor usage relative to oil usage.

With the equilibrium real wage in hand, I can use (11) to obtain the equilibrium level of oil-sector labor in the economy as well as the equilibrium value of oil production

$$N_t^o = \left[\frac{\nu Z_t}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \left(\frac{P_t^O}{A_t} \right)^{\frac{1}{\alpha}} \right]^{\frac{1}{1-\nu}}, \tag{20}$$

$$O_t^S = \left[\frac{\nu Z_t^{\frac{1}{\nu}}}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \left(\frac{P_t^O}{A_t} \right)^{\frac{1}{\alpha}} \right]^{\frac{\nu}{1-\nu}}. \tag{21}$$

An increase in oil prices provides an incentive for the oil sector to produce more, therefore creating a higher demand for labor. This is reflected by equation (20). The direction of the effect of an increase in P_t^O on N_t^o is the same as that of an increase in Z_t and can also be interpreted as an increase in productivity. This increase in N_t^o also implies an increase in oil production, as can be seen from (21). Additionally, the equilibrium level of labor in the final good sector, N_t, decreases when oil prices increase.

The overall effect on the equilibrium level of total labor in this economy is also negative. This is due to the fact that N_t is reduced when P_t^O increases and also to the fact that there is a wealth effect caused by increased oil revenues, which implies that it is optimal for the household to lower total labor supply. The sum of these two effects dominates the increase in N_t^o, therefore implying an overall decrease in the equilibrium level of total labor. Intuitively, there is labor mobility from the final good sector to the oil sector when the price of oil increases. This indicates that the economy will tend to reallocate relatively more labor resources to the relatively more profitable oil sector, therefore decreasing equilibrium levels of labor in the final good sector. This result, together with the fact that labor-intensive oil is decreasing in P_t^O, implies that final good sector output, Y_t, responds negatively to increases in P_t^O.

A natural question that arises at this point is whether, in general, welfare is increasing in P_t^O. The answer to this inquiry is that, as long as there is an equilibrium with positive oil exports, an increase in the real price of oil (P_t^O) increases welfare[4]. Note that this conclusion is in line with the standard result from international trade which affirms that an increase in the terms of trade improves welfare. For example, Svensson (1984) provides a detailed analysis of the welfare and terms of trade effects of oil price increases. The result of this model is in agreement with his conclusion for oil exporting countries. Clearly, the absence of oil exports in this model means that an increase in the real price of oil implies a deterioration in the terms of trade. In this situation, the effect on welfare would be negative. The following proposition summarizes these observations.

Proposition 2 *The static economy satisfies the following conditions with respect to changes in the real price of oil:*

[4] An additional requirement is, of course, that marginal utility of consumption be strictly positive.

i) real wages fall: $\frac{\partial W_t}{\partial P_t^O} < 0$,

ii) labor-intensive oil falls: $\frac{\partial o_t}{\partial P_t^O} < 0$,

iii) labor in the oil sector rises: $\frac{\partial N_t^O}{\partial P_t^O} > 0$,

iv) total oil production increases: $\frac{\partial O_t^S}{\partial P_t^O} > 0$,

v) both labor in the final goods sector and total labor fall: $\frac{\partial N_t}{\partial P_t^O} < 0$ *and* $\frac{\partial N_t^S}{\partial P_t^O} < 0$,

vi) final goods production falls: $\frac{\partial Y_t}{\partial P_t^O} < 0$,

vii) oil exports increase: $\frac{\partial O_t^*}{\partial P_t^O} > 0$,

viii) welfare increases: if $O_t^* > 0$, *then* $\frac{\partial U_t}{\partial P_t^O} > 0$.

Note that property vii) reflects the economy's incentive to increase oil sales to the rest of the world when P_t^O increases. Additionally, it is important to point out that, until now, I have made no specific assumptions on the form of the utility function (1). Clearly, given its general form, a complete closed form solution for this economy does not exist. However, exact solutions exist for particular cases of the utility function and for first order loglinear approximations of the model. I explore solutions for a specific form of (1). In particular, in order to illustrate the distinct and specific consequences that an oil price shock can have on the economy, I study utility functions that belong to the class of isoelastic, separable functions:

$$U_t = \frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \frac{\left(N_t^S\right)^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}}. \tag{22}$$

The parameter $\sigma \geq 0$ denotes the intertemporal elasticity of substitution, and $\varphi \geq 0$ represents the Frisch elasticity of labor supply. The following two special cases elaborate on the properties of the model. Specifically, I will study the different conclusions that can be obtained with different parameter values of the elasticity of labor supply.

A first case is obtained by setting the intertemporal elasticity of substitution to unity ($\sigma = 1$) and by assuming an infinitely elastic labor supply ($\varphi = \infty$). The utility function in this scenario is then given by

$$U_t = \ln C_t - N_t^S \tag{23}$$

The following proposition illustrates the properties of the solution to this case as well as the sensitivity of the economy to changes in the price of oil. The proof is given in Appendix B.

Proposition 3 *If $\sigma = 1$ and $\varphi = \infty$ in equation (22), then the competitive equilibrium of this economy has the following properties:*

i) $C_t = W_t$,

ii) when the real price of oil increases, consumption and real wages fall: $\frac{\partial C_t}{\partial P_t^O} = \frac{\partial W_t}{\partial P_t^O} < 0$,

iii) $N_t^o < \nu < 1$ and $N_t < \alpha < 1$.

Property i) states that the consumption level in this economy equals the wage level, and is a direct consequence of the form of the utility function. Property ii), together with the fact that $\frac{\partial N_t}{\partial P_t^O} < 0$ and $\frac{\partial Y_t}{\partial P_t^O} < 0$, describe the contractionary effects of an increase in the price of oil. It is important to emphasize that, in equilibrium, wages respond negatively to an increase in P_t^O since real marginal cost has to be brought back to equal unity, or equivalently, because it decreases marginal productivity of labor in the final goods sector. Moreover, the contractionary effect on consumption C_t is caused by the negative response of the final goods sector to an increase in the price of oil, a response that dominates the increase in lump sum consumption $P_t^O O_t^*$ caused by the same oil price increase. Property iii) states that the equilibrium levels of labor in both sectors is less than unity.

As a second case, I now analyze the scenario where labor supply is perfectly inelastic. For this I assume, without loss of generality, that $N_t^S = 1$. The utility function in this situation is given by

$$U_t = \frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - 1 \tag{24}$$

The following proposition summarizes the main properties obtained from this case as well as the responsiveness of the economy to increases in the price of oil. The details of the proof are presented in Appendix B.

Proposition 4 *If $N_t^S = 1$ in (22), then the competitive equilibrium of this economy has the following properties:*

i) consumption increases when the real price of oil increases: $\frac{\partial C_t}{\partial P_t^O} = O_t^* > 0$,

ii) in order for the final goods sector to exist, it must be true that $P_t^O < \frac{W_t}{\nu Z_t}$*, which is equivalent to the condition* $N_t^o < 1$.

The main distinction between Case 1 and Case 2 is that in the latter economy consumption C_t increases whenever there is an increase in P_t^O. This is a consequence of two facts: one, the positive relationship between O_t^* and P_t^O, which dominates the negative effect in the production of the final good, caused by an increase in P_t^O; and two, given that labor supply is perfectly inelastic, the household is forced to provide a predetermined amount of labor. Hence, the household will shift labor resources from the final goods sector to the more profitable oil sector, while maintaining total labor supply constant. Therefore, compared to Case 1, the decrease in N_t, and therefore in $O_t = o_t N_t$, produces a decrease in Y_t that cannot compensate the increase in $\Omega_t = P_t^O O_t^*$. From the fact that $C_t = Y_t + \Omega_t$ we conclude that the overall effect on C_t caused by an increase in P_t^O is positive.

These two cases have shown that an increase in oil prices can have distinct effects on an economy depending, mainly, on the elasticity of labor supply. This set of results also indicates the existence of a tension between the contractionary effects of an increase in the price of oil, and the wealth effect generated by it. The dominant force in this tension is determined by the economy's flexibility to reallocate labor resources between sectors.

3.7 Adding government

As previously mentioned, fiscal policy in most oil-producing economies is inherently dependent on oil profits. In this section, I add to the static model a government sector that taxes oil sector profits in order to finance its spending. To do this, I assume that government expenditures, G_t, enter the utility function of the household in the following way:

$$U_t^G = U_t + h(G_t), \tag{25}$$

where $h(\cdot)$ is strictly increasing and concave. Additionally, the household's budget constraint now becomes:

$$S_t^{\tau} \geq 0, \tag{26}$$

where $S_t^{\tau} \equiv S_t - \tau \Pi_t^O$ denotes after tax savings, and $\tau \in [0, 1]$ is a proportional tax rate on oil profits. The government has to balance its budget in the static economy, so that $G_t = \tau \Pi_t^O$.

The effects of an increase in the price of oil on consumption and government structure depend on the fiscal policy regimes. I explore two alternatives. First, I consider the scenario where the government desires to keep its spending at a fixed level $\bar{G}$, with τ adjusting so as to satisfy this restriction. In this case, the resource constraint becomes $C_t = Y_t + \Omega_t - \bar{G}$, which implies that increases in the price of oil have no effect on consumption other than the ones described previously for an economy without government spending. Moreover, as expected, $\tau = \frac{\bar{G}}{\Pi_t^O}$ is a decreasing function of P_t^O, which implies that when the price of oil decreases, the tax rate increases in order to keep government spending at its constant level $\bar{G}$. In this situation, τ plays the role of a fiscal stabilizer.

As a second fiscal policy regime, I consider the case of a fixed tax rate $\bar{\tau}$, and assume that U_t is defined as in (1). The following result holds under these assumptions.

Proposition 5 *Under a fixed tax rate regime, with U_t defined as in (1), $\exists\ \tau'$ such that $\forall$ $\bar{\tau} \lesseqgtr \tau'$, $\frac{\partial C_t}{\partial P_t^O} \gtreqless 0$.*

This result states that it is possible to have a sufficiently high tax rate that causes consumption to decrease whenever oil prices increase. Similarly, since Π_t^O is increasing in P_t^O, we also have that $\frac{\partial G_t}{\partial P_t^O} > 0$.

This finalizes the analysis of the static model. With this intuition in hand, we now proceed to construct the more realistic dynamic model.

4 The Model

In this section, I present the dynamic stochastic general equilibrium model. Specifically, I extend the setup described for the static model by adding an infinite horizon and a monetary authority. Additionally, I assume there is a continuum of final good firms each producing a specific variety. These variety producers coexist in an environment of monopolistic competition and set prices as in Calvo (1983). The remainder of this section provides the explicit description of the dynamic model.

The economy is inhabited by a representative household that derives utility from consumption and disutility from labor. The representative household seeks to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t, \tag{27}$$

subject to the following sequence of budget constraints:

$$B_t \leq (1 + i_{t-1}) B_{t-1} + S_t, \quad t \geq 0, \tag{28}$$

where U_t and S_t are defined in (22) and (3), respectively. The household enters period t with nominal bond holdings B_{t-1} and savings S_t. It receives interest payments $i_{t-1}B_{t-1}$, where i_{t-1} denotes the nominal interest rate, and carries B_t in bonds to the next period. The parameter $\beta \in (0, 1)$ is the discount factor. A standard nonsatiation argument guarantees that, in equilibrium, (28) will hold with equality.

Now, C_t refers to a Dixit-Stiglitz CES index of consumption of final goods:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{29}$$

where $C_t(j)$ is the household's demand for differentiated good j, and θ is the price elasticity of demand for variety j. The household's optimal allocation for a given level of expenditure yields the following demand function for variety j:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t, \tag{30}$$

where $P_t(j)$ is the nominal price of a unit of $C_t(j)$, and P_t, the consumption price index, is defined as

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \tag{31}$$

The household maximizes (27) subject to (28). The first order conditions for this optimization problem are then given by

$$\beta \mathbb{E}_t \left[\frac{C_{t+1}^{-\frac{1}{\sigma}}}{C_t^{-\frac{1}{\sigma}}} \frac{P_t}{P_{t+1}}\right] = \frac{1}{1+i_t}, \tag{32}$$

$$\frac{\left(N_t^S\right)^{\frac{1}{\varphi}}}{C_t^{-\frac{1}{\sigma}}} = \frac{W_t}{P_t}, \tag{33}$$

and the stochastic discount factor between periods t and $t+k$ is defined as

$$\Xi_{t,t+k} \equiv \beta^k \frac{C_{t+k}^{-\frac{1}{\sigma}}}{C_t^{-\frac{1}{\sigma}}} \frac{P_t}{P_{t+k}}. \tag{34}$$

There is now a continuum of unit mass of final good variety producers. In each period t, a typical firm j produces a differentiated good using the technology described in (5). The oil sector is identical to the one described in the static model with the additional assumption that $\ln Z_t$ follows an AR(1) process defined by

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_{Z,t}. \tag{35}$$

Similarly, I assume an analogous AR(1) process for $\ln A_t$:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}. \tag{36}$$

Here, $\rho_Z, \rho_A \in [0, 1)$ and $\varepsilon_{Z,t}, \varepsilon_{A,t}$ are independent and identically distributed random variables with mean zero and finite standard deviation σ_Z and σ_A, respectively. Finally, I denote the real price of oil by $Q_t \equiv \frac{P_t^O}{P_t}$.

4.1 Price Setting

Final good producing firms set prices as in Calvo (1983). Each period, a typical firm sets a new price with probability $1 - \phi$, independently of the time that has passed since the firm last changed its price, and keeps its current price with probability ϕ. Those firms that are to change their price do so by choosing the price $\bar{P}_t(j)$ so as to maximize the present discounted value of the flow of profits according to the following problem:

$$\max_{\bar{P}_t(j)} \sum_{k=0}^{\infty} \phi^k \mathbb{E}_t \left\{ \Xi_{t,t+k} Y_{t+k}(j) \left[\bar{P}_t(j)(1+\zeta) - MC_{t+k} \right] \right\} \tag{37}$$

subject to

$$Y_{t+k}(j) = \left(\frac{\bar{P}_{t+k}(j)}{P_{t+k}} \right)^{-\theta} \left(C_{t+k} - \frac{P_{t+k}^O O_{t+k}^*}{P_{t+k}} \right), \tag{38}$$

where ζ is a subsidy to producers of final goods. Note that in (38), C_{t+k} is the total demand for the composite final good in period $t+k$. This final good is either produced in the economy or obtained by transforming the oil revenues from oil exports into their final consumption good equivalent, $\frac{P_{t+k}^O O_{t+k}^*}{P_{t+k}}$. The total demand faced by the final good sector is therefore $Y_{t+k}^d \equiv C_{t+k} - \frac{P_{t+k}^O O_{t+k}^*}{P_{t+k}}$.

In (37), MC_{t+k} is the period $t+k$ nominal marginal cost in and is given by

$$MC_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{W_t^\alpha \left(P_t^O \right)^{1-\alpha}}{A_t}. \tag{39}$$

The first order condition for a typical firm implies

$$\bar{P}_t(j) = \bar{P}_t = \frac{\theta}{(\theta - 1)(1+\zeta)} \frac{\sum_{k=0}^{\infty} (\phi\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\frac{1}{\sigma}} P_{t+k}^{\theta-1} Y_{t+k}^d MC_{t+k} \right]}{\sum_{k=0}^{\infty} (\phi\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\frac{1}{\sigma}} P_{t+k}^{\theta-1} Y_{t+k}^d \right]}. \tag{40}$$

Since all firms resetting prices in any given period are identical, they will choose the same price; we therefore drop the j subscript from this point on and write the optimal price chosen by all firms as $\bar{P}_t$. Using equation (31) and the assumptions of the Calvo price setting model one can then write the associated law of motion for P_t as

$$P_t = \left[\phi P_{t-1}^{1-\theta} + (1-\phi)\bar{P}_t^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{41}$$

4.2 Market clearing conditions and resource constraint

The output aggregator in this economy is given by

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \tag{42}$$

Additionally, market clearing conditions for oil and labor are given by (15) and (17), respectively, with $O_t \equiv \int_0^1 O_t(j)dj$ and $N_t \equiv \int_0^1 N_t(j)dj$. The resource constraint of the economy is again equation (13).

Finally, I assume that bonds are in zero net supply so that, in equilibrium, $B_t = 0$. This assumption can be justified if we take into account that EMEs have borrowing constraints, or cannot borrow as much, as developed economies. For instance, as pointed out by Hausmann (2004), EMEs face considerable difficulties to borrow abroad in their own currency and are, to a great extent, restricted to short-term borrowing in foreign currency. This simplification is only a first step in the analysis and, in principle, one should consider relaxing it by introducing a capital account in the model. By allowing this additional feature, the model would be considering the fact that some oil-producing countries (e.g., Norway) are net savers.

This finalizes the setup of the dynamic model. The following sections present the main results derived from this framework.

4.3 The Phillips Curve

Log-linearization of (39), (40) and (41) around the deterministic steady state allows us to write the Phillips curve for this economy in a well known form:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \widehat{mc}_t, \tag{43}$$

where, $\kappa = \frac{(1-\phi)(1-\beta\phi)}{\phi}$, $mc_t \equiv \frac{MC_t}{P_t}$ is the real marginal cost, and the notation $\widehat{h}_t \equiv \ln H_t - \ln H$ denotes the log deviation from the steady state for the variable H_t.

In a standard model, $\widehat{mc}_t$ depends only on final goods output $\hat{y}_t$ and productivity $\hat{a}_t$. In an oil-producing economy, oil prices have additional effects on inflation. In particular, the following proposition shows that, in this economy, not only the standard cost-push effects of an oil price shock are present, but additional inflationary pressures exist due to the presence of a wealth effect that perturbs the marginal cost.

Proposition 6 *The Phillips curve for this economy can be rewritten as*

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(k_1 \hat{y}_t - k_2 \hat{a}_t + k_3 \hat{q}_t + k_4 \hat{z}_t + k_5 \hat{\Omega}_t) \tag{44}$$

where $k_i > 0$ *for* $i = 1, 2, ..., 5$.

Proof. *See Appendix B.* ■

The first two terms inside the parenthesis in equation (44) involve the usual effects on inflation encountered in a standard Phillips curve. The first term implies that an increase in output generates inflationary pressures due to the increase in real wages, and hence in real marginal cost, that it causes. The second term is a direct consequence of (39) and of the fact that, *ceteris paribus*, labor demand in the final good sector decreases whenever there is an increase in productivity in the final good sector. The third term is the cost-push effect of oil prices. An increase in the price of oil has a direct positive effect on marginal cost and, therefore, on inflation. The fourth term shows that an increase in productivity in the oil sector implies an increase in inflation. The reason for this is that, when oil sector productivity increases, the marginal product of labor in this sector increases too. This is equivalent to a shift in labor demand in the oil sector. Given the assumption of decreasing returns to scale in the oil sector technology, this results in an increase in both the equilibrium level of wages and oil sector labor, which in turn implies an increase in marginal costs. Finally, the fifth term inside the parenthesis can be interpreted as the positive wealth effect of the extra consumption generated by an oil price shock on an oil-producing economy. This wealth effect incentivizes the household to reduce their labor supply therefore increasing the equilibrium level of wages. The final consequence is an increase in marginal cost and inflation.

Evidently, if $k_3 = k_4 = k_5 = 0$ we are back to the standard Phillips curve. This case is obtained if we assume that there is no oil sector.

4.4 Taylor rule

For the monetary authority, I will assume a generalized Taylor rule of the form

$$\hat{\imath}_t = \phi_\pi \pi_t + \phi_y(\hat{y}_t - \hat{y}_t^n) + \phi_c(\hat{c}_t - \hat{c}_t^n), \tag{45}$$

where $\hat{\imath}_t \equiv \ln\left(\frac{1+i_t}{1+i}\right)$, $\phi_\pi > 0, \phi_i \geq 0$ for i=y, c, and $\phi_y \phi_c = 0$. The reason for including the term $\phi_y(\hat{y}_t - \hat{y}_t^n)$ is mainly because I would like to explore the implications of a monetary authority reaction to a measure of output that only includes the final goods sector. On the other hand, it is only natural to assume that the central bank's interest rate rule reacts only to $\phi_c(\hat{c}_t - \hat{c}_t^n)$, which in this model is a measure of the gross output gap. To see this, one can think of C_t as having two components: Y_t and Ω_t. In other words, C_t is a measure of aggregate output in the economy, since it includes not only Y_t but also the consumption good equivalent of oil production devoted to exports. The first component is not directly linked to the volatility of oil prices. However, Ω_t depends directly on the price of oil and, given that a predetermined amount of oil exists, it is subject to its volatile behavior. Hence, given the nature of the oil sector, one can plausibly justify an interest rate rule which only considers the output gap in the final goods sector.

I will therefore provide impulse response analysis to the following Taylor rules. First, I will assume a standard Taylor rule ($\phi_y = 0$). The second case will consist of $\phi_c = 0$. Following Cúrdia (2005), the values assumed for ϕ_π and ϕ_y are 3 and 1/3 respectively, which are consistent with emerging market economies. Note that, since $\hat{c}_t = \frac{Y}{C}\hat{y}_t + \frac{Q\Omega}{C}\hat{\Omega}_t + \frac{Q\Omega}{C}\hat{q}_t$, we have that $\phi_c\hat{c}_t = \phi_c\frac{Y}{C}\hat{y}_t + \phi_c\frac{Q\Omega}{C}\hat{\Omega}_t + \phi_c\frac{Q\Omega}{C}\hat{q}_t \equiv \phi_y\hat{y}_t + \phi_\Omega\hat{\Omega}_t + \phi_q\hat{q}_t$. Hence, a value of $\phi_c = .5$ implies values of ϕ_y, ϕ_Ω, and ϕ_q of 1/3, .17 and .17, respectively, and according to steady state values of Y, C and $Q\Omega$.

4.5 Loss Function

This model allows the derivation of a natural and inherent measure of welfare that can be used to explore optimal monetary policy. In particular, by writing a second order approximation to the household's utility function, one can obtain the associated loss function for this economy. The fact that the economy consists of two sectors implies a nonlinearity that prevents us from solving for optimal monetary policy for a general case. I therefore specialize to the case for which the disutility in labor is linear ($i.e., \varphi = \infty$), and I assume that the subsidy for final good producers exactly offsets the markup distortions that are present. In this way, the flexible price equilibrium will be at its efficient level. The following proposition specifies the form of the period loss function.

Proposition 7 *The associated period loss function for this economy when $\varphi = \infty$ is given by*

$$L_t = \lambda_1\pi_t^2 + \lambda_2\hat{x}_t^2 \tag{46}$$

where $\hat{x}_t \equiv \hat{c}_t - \hat{c}_t^n$, $\hat{c}_t^n$ is the efficient level of consumption under flexible prices, and the λ_i's are functions of the model's fundamental parameters.

Proof. *A detailed derivation of the loss function appears in Appendix B.* ■

5 Benchmark Calibration and Impulse Responses

The baseline calibration of the model is done to fit quarterly data for a representative oil-producing economy[5]. I assume that the discount factor $\beta = .977$, which is consistent with a steady state annualized interest rate of 9.7%. This value is the 2005 average[6] of the corresponding interest rates for the countries in Table 1. The intertemporal elasticity of substitution, σ, is set to equal 1/2 and the Frisch elasticity of labor supply, φ, is set to 1. These values for σ and φ are in agreement with those used by Devereux, Lane and Xu (2005) for emerging market economies. Castillo, Montoro and Tuesta (2005) also use the same value for σ. For the final good producing firms, the probability of resetting prices, $1 - \phi$, is set to equal .25, which means that, on average, firms will reset prices once a year. This implies a value of $\phi = .75$. The price elasticity of demand for consumption goods, θ, is set to 11, which implies a steady state monopolistic markup value of 10%. For the share of oil in the production function, $1 - \alpha$, I use a value of .13 which is in line with that used by Castillo et al. (2005). I set the parameter ν to .6 in the oil production function. This assumption is in agreement with the findings of García-Verdú (2005) for labor shares. In addition, the entire model is calibrated so that the steady state value of $\chi \equiv \frac{P_t^O O_t^*}{P_t Y_t + P_t^O O_t^*}$, the share of oil exports to GDP, equals .336, which is around the 2004 average of the corresponding shares presented in Table 1. For the technology processes, the autocorrelation parameter values are $\rho_A = \rho_Z = .9$, as in Medina & Soto (2005) and close to the values used by Leduc & Sill (2004). I assume a value of $\sigma_A = \sigma_Z = .017$ also in line with Medina & Soto (2005).

For the first set of results, I consider and exogenous process for the real price of oil (Q_t). In particular, I assume that it evolves according to the following AR(1) rule:

$$\ln Q_t = \rho_o \ln Q_{t-1} + \varepsilon_{o,t} \tag{47}$$

Following Medina and Soto (2005), I set $\rho_o = .88$ and assume a standard deviation for $\varepsilon_{o,t}$

[5] A table summarizing the calibrated parameters appears in Appendix A.

[6] Not including Iraq.

of $\sigma_o = .134$. These values are also very close to those used by Castillo et al. (2004). The following sections present the responses of the relevant variables in the economy to a shock in the price of oil.

5.1 Responses to an oil price shock with O_t^* determined endogenously.

Figure 1 illustrates the impulse response functions of an exogenous increase in the real price of oil. Under a Taylor rule that targets inflation π_t and the output gap in the final good sector $\hat{y}_t - \hat{y}_t^n$, we note that an oil price shock provides an incentive to the oil sector therefore increasing total oil supply and oil exports, and hence equilibrium level of oil sector labor. These effects imply an increase in $\Omega_t = Q_t O_t^*$ as well. For the final good sector, we notice that the oil price shock decreases the equilibrium level of oil O_t given that it lowers oil demand. Labor in the final goods sector, N_t, also decreases for two reasons. First, the increase in oil prices reduces marginal productivity of labor in this sector therefore requiring a reduction in labor demand; and second, there is migration to the now more productive oil sector. Additionally, there is a wealth effect induced by the increase in oil prices that reduces labor supply from the household. In equilibrium, total labor decreases while oil sector labor raises and final good sector labor is lowered. The net effect of these forces on the labor market is an increase in real wages mainly because the decrease in labor demand is lower than the one in labor supply. As a consequence of the reductions in O_t and N_t , there is a decline in Y_t. Moreover, the overall effect on marginal cost of the higher oil prices is positive. This increases the level of inflation in the economy. The net effect on consumption of the reduction in Y_t and the increase in Ω_t is an increase in C_t. This reflects that the dominant force in the economy after an oil price shock is provided by the positive effect in Ω_t. The increase in Ω_t more than compensates the contraction in output in the final good sector.

I now turn to the case where the monetary authority targets inflation and the consumption gap $\hat{c}_t - \hat{c}_t^n$. That is, I now assume that the coefficients in (45) take the values $\phi_\pi = 3, \phi_c = .5$

and $\phi_y = 0$. The main distinction between this case and the former one, in which the central bank targets $\hat{y}_t - \hat{y}_t^n$, is that the reaction of the central bank to $\hat{c}_t - \hat{c}_t^n$ produces a greater contraction in final goods output Y_t. In turn, this implies a reduction in N_t that is more pronounced, implying a higher level of migration to the oil sector as well. The net effect of these forces on the labor market is a reduction in real wages mainly because the decrease in labor demand is greater than the one in labor supply. Now, the overall effect on marginal cost of the higher oil prices is negative. This decreases the level of inflation in the economy. Moreover, the level of C_t is now lower than in the previous scenario because the reduction in Y_t is higher.

It is important to mention that this economy can be reduced so as to explain the qualitative effects of oil price shocks on net oil importers. In particular, if I were to start with a sufficiently small steady state value for $\frac{P_t^O O_t^*}{P_t Y_t + P_t^O O_t^*}$, I would obtain a contraction in output and an increase in inflation when the economy faces an increase in oil prices.

Figure 2 plots the impulse response functions to a productivity shock in the oil sector. Under both Taylor rules, the results are, as expected, a positive effect on oil production, oil exports, and oil sector labor. The increase in O_t^* implies an increase in Ω_t and hence a positive wealth effect, which is translated into lower work effort from the household. The now more productive oil sector attracts workers from the final good sector, which causes a decline in N_t. In turn, marginal productivity of oil in the final good sector declines, and for a fixed level of real oil prices the level of oil usage O_t must decline too. The joint decline in O_t and N_t produce a decline in Y_t. The net effect of the changes in Y_t and Ω_t on C_t is positive, which implies that the increase in Ω_t overcomes the decrease in Y_t that is generated by the higher oil price.

On the other hand, the effects on wages and inflation are dependent on the type of Taylor rule that is specified. For the case in which the Taylor rule reacts to $\hat{c}_t - \hat{c}_t^n$, the final good sector is allowed to contract by a larger amount than in the case with a Taylor rule that responds to $\hat{y}_t - \hat{y}_t^n$. The results on wages are similar to those encountered under an oil price shock. Hence, inflation increases in the case characterized by a Taylor rule that cares about Y_t, and decreases by a negligible amount in the other scenario.

Not surprisingly, the qualitative effects of an increase in oil sector productivity Z_t are identical to the corresponding effects implied by a rise in the real price of oil. This is true precisely because an increase in the price of oil can be interpreted, to a certain degree, as an increase in oil sector productivity.

The responses of the relevant variables to an increase in productivity in the final good sector, A_t, are shown in Figure 3. We notice that an increase in A_t makes the final good sector relatively more productive than the oil sector. There is labor mobility from the oil sector towards the final good sector which causes marginal productivity of oil in this sector to increase therefore requiring an increase in O_t for a fixed level of the real oil price. Since A_t, O_t and N_t are higher, Y_t increases as well. The lower level of N_t^o implies a reduction in oil production O_t^S. The joint effect of the decrease in O_t^S and the increase in O_t implies a reduction in O_t^* and therefore in Ω_t. However, as one would expect, the net effect of the productivity shock on consumption and welfare is positive. We can also mention that wages are raised given the fact that the income effect that goes with the higher level of consumption implies a decrease in labor supply that is larger than the decrease in overall labor demand. The net effect on marginal cost and inflation of the higher levels of real wages and productivity is negative.

As a final point, one could also think of modeling the effects of an oil price shock by considering O_t^* as an exogenous process and letting Q_t, the real price of oil, to be determined endogenously. This exercise shows that that the qualitative effects are the same as those obtained by treating Q_t as an exogenous process. In this model, this is true given the fact that an increase in foreign demand for oil implies an increase in the real price of oil.

5.2 Optimal Monetary Policy

To solve for optimal policy, the monetary authority maximizes the household's loss function with respect to π_t and $\hat{x}_t$ subject to (44). The Lagrangian in this case is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} (1-\beta)\beta^t \left\{ \left[\lambda_1 \pi_t^2 + \lambda_2 \hat{x}_t^2\right] + \Theta_t \left[\beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa\alpha}{\sigma}\widehat{x}_t - \pi_t\right]\right\}.$$

The consolidated first order condition can be written as $\pi_t + \frac{\lambda_2}{\lambda_1}\frac{\sigma}{\kappa\alpha}(\hat{x}_t - \hat{x}_{t-1}) = 0$. It is well known that the solution to this problem implies that the optimal monetary policy requires complete stabilization of the output gap. From the constraint in the Lagrangian (i.e. the Phillips curve), we conclude that optimal policy also requires that inflation be stabilized. Hence, both output and inflation stabilization can be attained by defining an appropriate output gap, that which obtains under the assumption of flexible prices, and which includes the consumption good equivalent of oil revenues, as well as the cost push effects of oil prices. Figure 4 plots the impulse responses to an oil price shock under optimal policy and under the Taylor rule $\hat{\imath}_t = 3\pi_t + .5(\hat{c}_t - \hat{c}_t^n)$. Note that the only noticeable difference between the superimposed plots is that under optimal policy, both inflation and output are completely stabilized and the interest rate's response is not as pronounced. A similar interpretation holds for Figures 5 and 6.

It is important to explore what the differences are, in terms of welfare, among Taylor-type rules and optimal policy. Table 2 illustrates the fact that the monetary authority should take into account the wealth effect in order for monetary policy to be significantly close to the optimal equilibrium. To compare across monetary regimes, I calculate the household losses as $\mathbb{E}[L_t] = \lambda_1 Var(\pi_t) + \lambda_2 Var(\hat{x}_t)$. Note that, since $C_t = Y_t + \Omega_t$, the Taylor rule shown in Table 2 is just decomposing C_t into its individual terms. With this in mind, we can compare the welfare gains that can be achieved by successively including these components.

Not surprisingly, a Taylor rule that reacts to consumption performs better than any of the preceding rules. This result is inherent to our derivation of the households loss function,

	$\hat{\imath}_t = \phi_\pi \pi_t + \phi_y(\hat{y}_t - \hat{y}_t^n) + \phi_\Omega(\hat{\Omega}_t - \hat{\Omega}_t^n) + \phi_q \hat{q}_t$				
	Loss	ϕ_π	ϕ_y	ϕ_Ω	ϕ_q
Taylor rule 1	348.4203	3	1/3	0	0
Taylor rule 2	12.4595	3	1/3	.17	0
Taylor rule 3	11.0545	3	1/3	.17	.17
$\hat{\imath}_t = 3\pi_t + .5\hat{x}_t$	.0131	-	-	-	-
Optimal policy	0	-	-	-	-

Table 2: Losses in the benchmark economy with linear disutility in labor.

for it is based on the assumption that the household cares about C_t and not specifically on Y_t. However, in an oil-producing economy it is possible for a monetary authority to care more about the volatility of Y_t relative to that of C_t. One possible reason for this is that economies that depend heavily on oil are subject to exogenous and volatile events that determine the final value of their oil production. Given that the value of oil exports is therefore exogenously determined, a central bank may be better off worrying about the less volatile sector Y_t . By reacting to $(\hat{y}_t - \hat{y}_t^n)$, the monetary authority can avoid pronounced fluctuations in the less volatile sector. Put another way, if we were to compare the volatilities implied by the Taylor rule $\hat{\imath}_t = 3\pi_t + .5\hat{x}_t$ with those obtained under the Taylor rule $\hat{\imath}_t = 3\pi_t + .5(\hat{y}_t - \hat{y}_t^n)$ we would conclude that both $Var(\hat{y}_t)$ and $Var(\hat{y}_t)/Var(\hat{x}_t)$ are smaller for the latter regime. Therefore, even though the former Taylor rule implies a lower variance in $\hat{x}_t$, the variance of $\hat{y}_t$ is higher.

6 Conclusion

The main objective of this paper is to study monetary policy in oil-producing economies that are characterized by having a significant dependence on oil revenues for their economic performance. Specifically, the model presented contributes to the New Keynesian literature by analyzing the effect of oil price shocks in a framework that explicitly models an oil sector. In addition to the standard contractionary and cost push effects, this New Keynesian general equilibrium model considers a new transmission channel for oil price shocks that can be described as an income effect that is generated by oil revenues. I first present a static model in which I illustrate that, when faced by an increase in oil prices, a tension may arise between its cost push and contractionary effects, and its implied income effect, which tends to increase consumption levels. The static model also shows that, as long as the equilibrium level of oil exports is positive, an increase in the price of oil is welfare improving. I then construct a dynamic model and I perform a calibration for a representative economy. I find that the New Keynesian Phillips curve includes a measure of oil income that is responsible for additional inflationary pressures. In particular, the model shows that the wealth effect caused by an increase in oil prices, has an effect on marginal cost and aggregate demand through its impact on wages. Impulse response functions show that, in terms of consumption and inflation stabilization, the economy responds better to a Taylor rule that reacts to consumption than to a Taylor rule that reacts to final goods production only, although in the former rule the volatility implied for non-oil output is higher.

I then turn to the study of optimal monetary policy. Specifically, the model allows a welfare-based approach to optimal monetary policy. To do this, I derive a loss function by writing the household's utility function as a second order Taylor expansion. I conclude that a central bank can stabilize both inflation and output without trade-off by reacting optimally to inflation and the consumption gap. Additionally, among Taylor-type rules, a rule that reacts to consumption and not only to final goods production is welfare superior.

Finally, although the model presented in this paper emphasizes an application to oil, it can, in principle, be employed to study monetary policy in economies that are dependent on natural resources.

7 References

Aoki, K, 2001. Optimal monetary policy responses to relative price changes. Journal of Monetary Economics 48, 55-80.

Aguiar-Conraria, L. and Y. Wen, 2005, Understanding the Impact of Oil Shocks, Federal Reserve Bank of St. Louis, Working Paper # 2005-042A.

Backus, D.K. and M.J. Crucini, 2000, Oil prices and the terms of trade, Journal of International Economics 50, 185-213.

Barsky, R.B. and L. Kilian, 2001, Do we really know that oil caused the Great Stagflation? A monetary alternative, in: B. Bernanke and K. Rogoff, eds.,NBER Macroeconomics Annual 16. 137-183.

Bernanke, B.S., M. Gertler and M.W. Watson, 1997, Systematic monetary policy and the effects of oil price shocks, Brookings Papers on Economic Activity, 91-157.

Bjørnland, H., 2000, The Dynamic Effects of Aggregate Demand, Supply and Oil Price Shocks - A comparative study. The Manchester School of Economic Studies, 68, 2000, 578-607

Bohi, Douglas R., 1989, Energy Price Shocks and Macroeconomic Performance.Washington, DC: Resources for the Future.

Burbidge, John, and Alan Harrison, 1984, "Testing for the Effects of Oil-Price Rises Using Vector Autoregressions." International Economic Review 25, 459–484.

Calvo, G., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics12 (3), 383-398.

Castillo, P., C. Montoro and V. Tuesta, 2005, Inflation Premium and Oil Price Volatility, Working Papers Central Bank of Chile 350, Central Bank of Chile.

Chang, J., 2005. Optimal Monetary Policy and Oil Supply Shocks- a New Keynesian Perspective, mimeo, Georgetown University.

Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: a new Keynesian perspective. Journal of Economic Literature 37, 1661-1707.

Clarida, R., J. Galí and M. Gertler, 2000, Monetary policy rules and macroeconomic stability: evidence and some theory, Quarterly Journal of Economics 115, 147-180.

Clarida, R., J. Galí and M. Gertler, 2001, Optimal monetary policy in open versus closed economies: an integrated approach, American Economic Review Papers and Proceedings 91, 248-252.

Cúrdia, V., 2005, Monetary Policy under Sudden Stops, Princeton University, Working Paper.

DeLong, J. Bradford, 1997, "America's Peacetime Inflation: The 1970s." In Reducing Inflation: Motivation and Strategy, edited by Christina D. Romer and David H. Romer, pp. 247–276. Chicago: University of Chicago Press.

Devereux, M., P. Lane and J. Xu, 2005, Exchange Rate Regime and Monetary Policy Rules for Emerging Markets, forthcoming, Economic Journal.

Finn, M.G., 2000, Perfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit, and Banking 32, 400-416.

Gali, J., Monacelli, T., 2005, Optimal monetary policy and exchange rate volatility in a small open economy. Review of Economic Studies 72, 707-734.

García-Verdú, R., 2005, Factor Shares from Household Survey Data, Dirección General de Investigación Económica, Banco de México, Working Paper 2005-05.

Gisser, Micha, and Thomas H. Goodwin, 1986, "Crude Oil and the Macroeconomy: Tests of Some Popular Notions." Journal of Money, Credit, and Banking 18, 95–103.

Hamilton, J., 1983, Oil and the macroeconomy since World War II, Journal of Political Economy 91, 228—248.

Hamilton, J., 1985, Historical causes of postwar oil shocks and recessions. Energy Journal 6, 97—116.

Hamilton, J., 1988a, A neoclassical model of unemployment and the business cycle, Journal of Political Economy 96, 593-617.

Hamilton, J., 1988b, Are the macroeconomic effects of oil-price changes symmetric? A comment, Carnegie-Rochester Series on Public Policy 28, 369-378.

Hamilton, J., 1996, This is what happened to the oil price-macroeconomy relationship, Journal of Monetary Economics 38, 215—220.

Hamilton, J., 2003, What is an oil shock?, Journal of Econometrics 113, 363—398.

Hamilton, J. and A. Herrera, 2004, Oil Shocks and Aggregate Macroeconomic Behavior: The Role of Monetary Policy, Journal of Money, Credit, and Banking, 36, 265—286.

Hooker, Mark A., 1999, "Are Oil Shocks Inflationary? Asymmetric and Nonlinear SpecificationsVersus Changes in Regime." Board of Governors of the Federal Reserve System working paper.

Hunt, Benjamin, Peter Isard, and Doug Laxton, 2001, "The Macroeconomic Effects of Higher Oil Prices," IMF Working Paper 01/14.

International Energy Agency, 2004, Analysis of the Impact of High Oil Prices on the Global Economy, May 2004.

Kamps, C. and C. Pierdzioch, 2002, Monetary Policy Rules and Oil Price Shocks, Kiel Working Paper No. 1090.

Kim, I.-M. and P. Loungani, 1992, The role of energy in real business cycle models,Journal of Monetary Economics 29, 173-189.

Leduc, S. and K. Sill, 2004, A quantitative analysis of oil-price shocks, systematic monetary policy, and economic downturns, Journal of Monetary Economics, 51, 781—808.

Loungani, P., 1986, Oil price shocks and the dispersion hypothesis, Review of Economics and Statistics 58, 536-539.

Medina, J. P., and C. Soto, 2005, Oil Shocks and Monetary Policy in an Estimated DSGE Model for a Small Open Economy, Central Bank of Chile Working Papers 353.

Mork, K. A., 1989, Oil and the macroeconomy when prices go up and down: An extension of Hamilton's results, Journal of Political Economy 91, 740—744.

Rasche, R. and J. Tatom, 1981, Energy price shocks, aggregate supply, and monetary policy: The theory and international evidence, Carnegie-Rochester Conference Series on Public Policy 14, 9-93.

Rotemberg, J.J. and M. Woodford, 1996, Imperfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit and Banking 28, 549-577.

Santini, Danilo J., 1985, "The Energy-Squeeze Model: Energy Price Dynamics in U.S. Business Cycles." International Journal of Energy Systems 5, 18–25.

Svensson, Lars E.O., 1984, "Oil Prices, Welfare, and the Trade Balance." Quarterly Journal of Economics 99, 4, 649-672.

Tatom, J., 1988, Are the macroeconomic effects of oil price changes symmetric? Carnegie-Rochester Conference on Public Policy 28, 325-368.

Tijerina-Guajardo, J and J. Pagán, 2003, Government Spending, Taxation, and Oil Revenues in Mexico, Review of Development Economics, 7(1), 152-164

Woodford, Michael, 2003, *Interest and Prices*, Princeton University Press, Princeton, New Jersey.

A Impulse Responses

Parameter	Value	Source
β	.977	$i = 9.7\%$ annual interest rate
σ	1/2	Devereux, Lane and Xu (2005), EMEs
φ	1	Devereux, Lane and Xu (2005), EMEs
$1-\phi$	.25	reset prices once a year
θ	11	10% SS markup
$1-\alpha$	.13	Castillo et al. (2005)
ν	.6	García-Verdú (2005)
$\frac{P_t^o O_t^*}{P_t Y_t + P_t^o O_t^*}$	.336	Average for countries in Table 1
ρ_A	.936	Medina & Soto (2005)
ρ_Z	.936	Medina & Soto (2005)
ρ_o	.88	Medina & Soto (2005)
σ_A	.017	Medina & Soto (2005)
σ_Z	.017	Medina & Soto (2005)
σ_o	.134	Medina & Soto (2005)

Table 3: Calibrated parameters for a representative oil-producing economy.

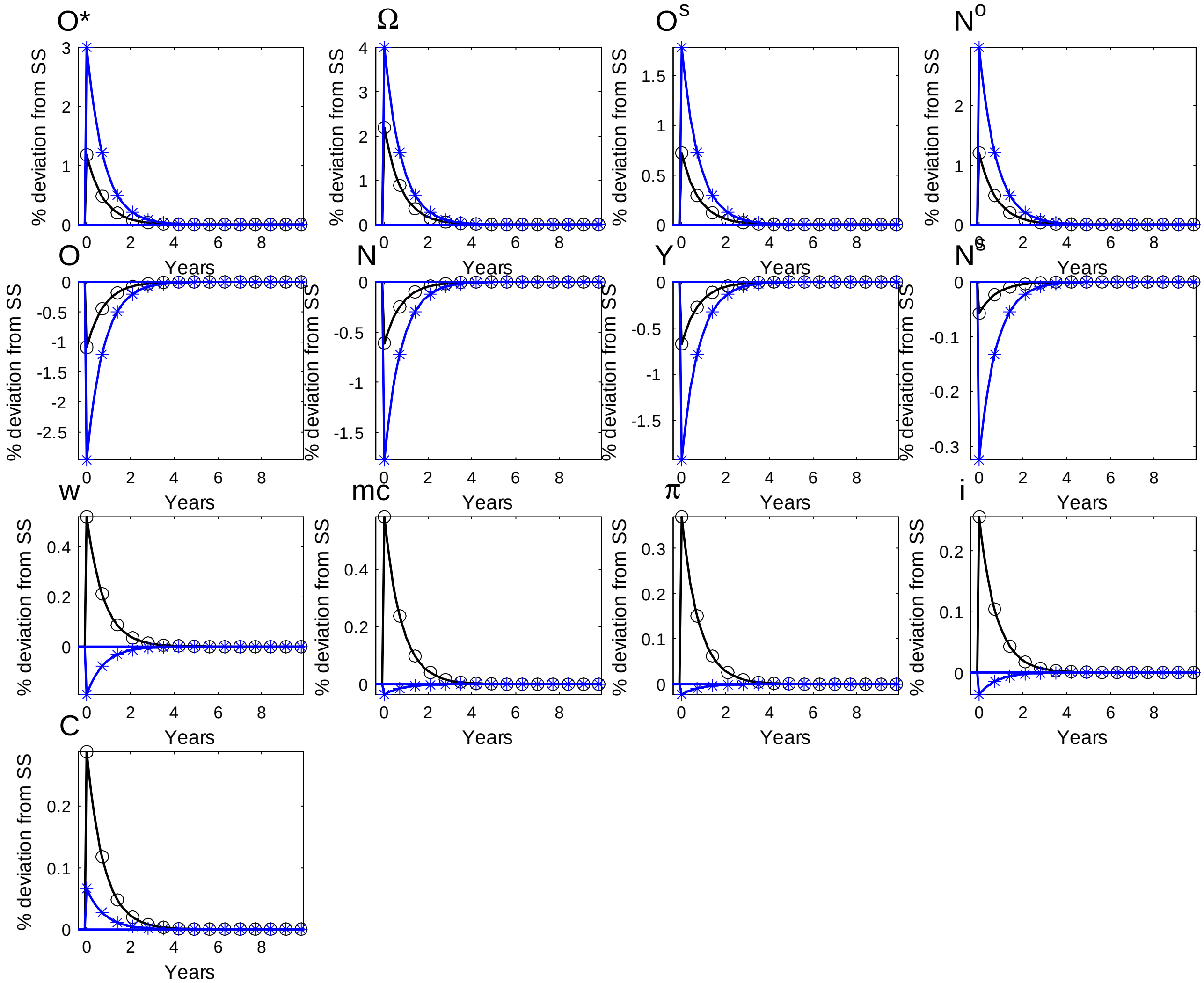

Figure 1: Responses to a shock in the real price of oil, with O^*_t determined endogenously. Black —○—: $i_t = 3\pi_t + 1/3(\hat{y}_t - \hat{y}^n_t)$. Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}^n_t)$.

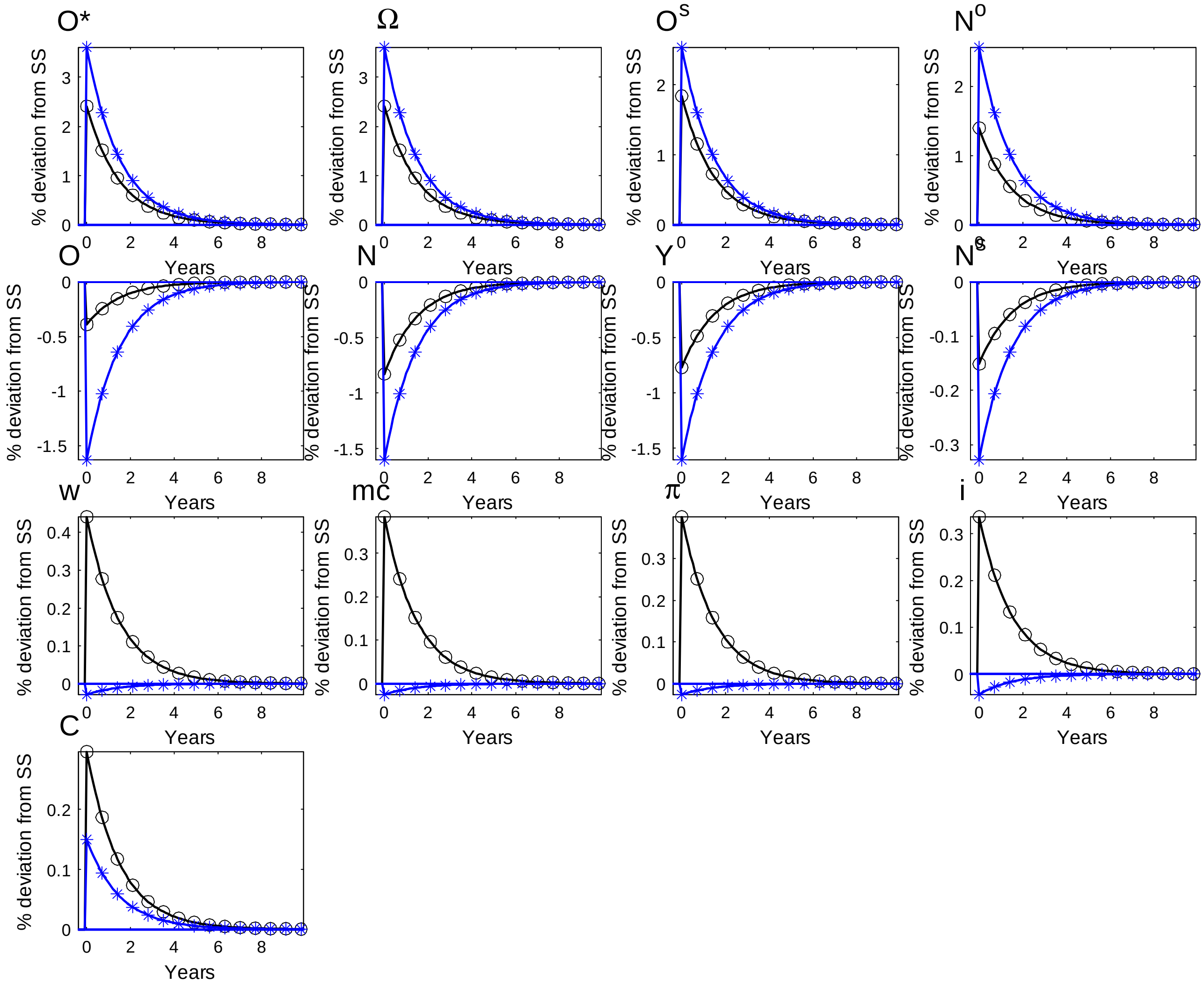

Figure 2: Responses to a productivity shock in the oil sector. Black —○—: $i_t = 3\pi_t + 1/3(\hat{y}_t - \hat{y}_t^n)$ Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}_t^n)$.

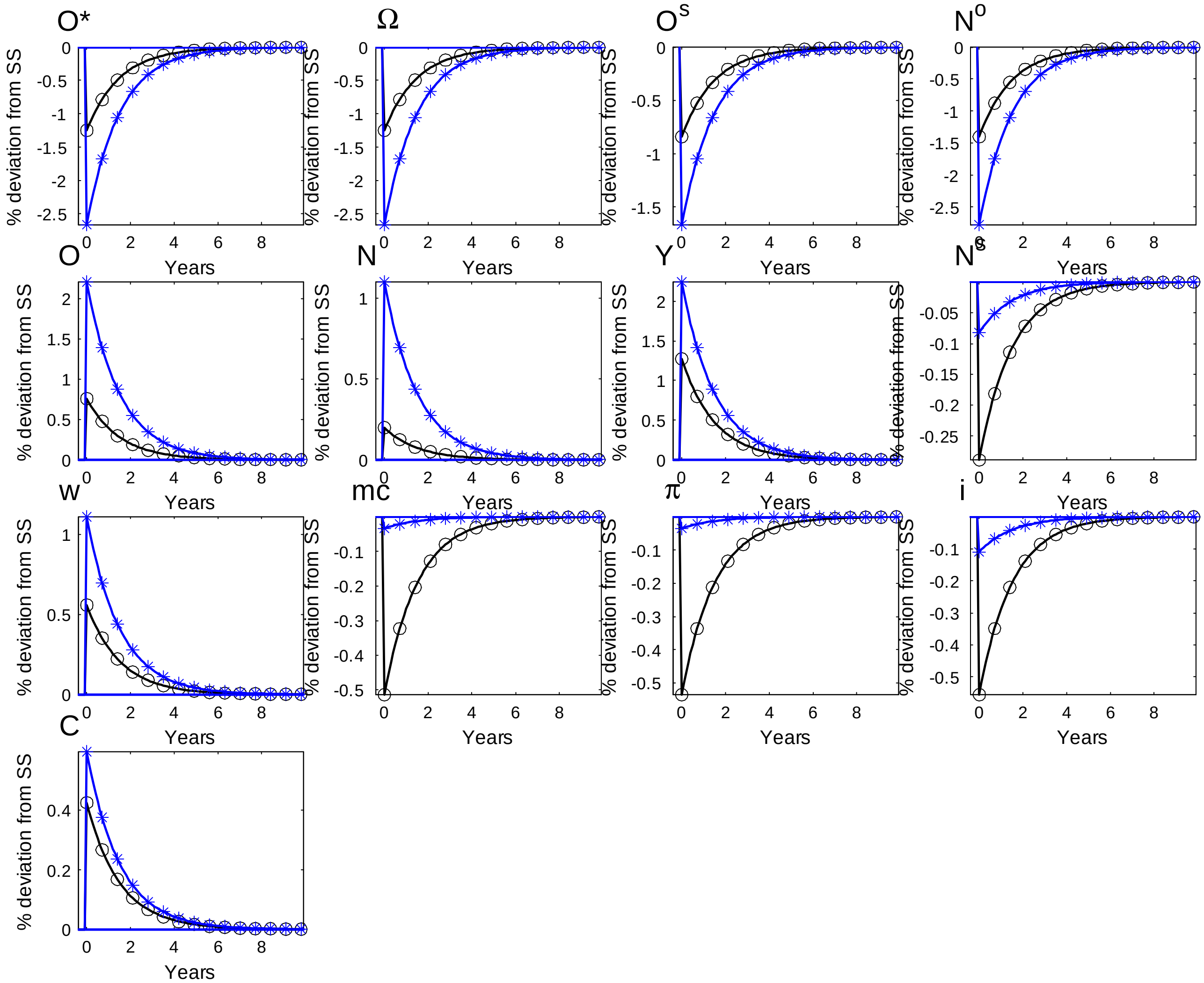

Figure 3: Responses to a productivity shock in the final goods sector. Black —○—: $i_t = 3\pi_t + 1/3(\hat{y}_t - \hat{y}_t^n)$ Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}_t^n)$.

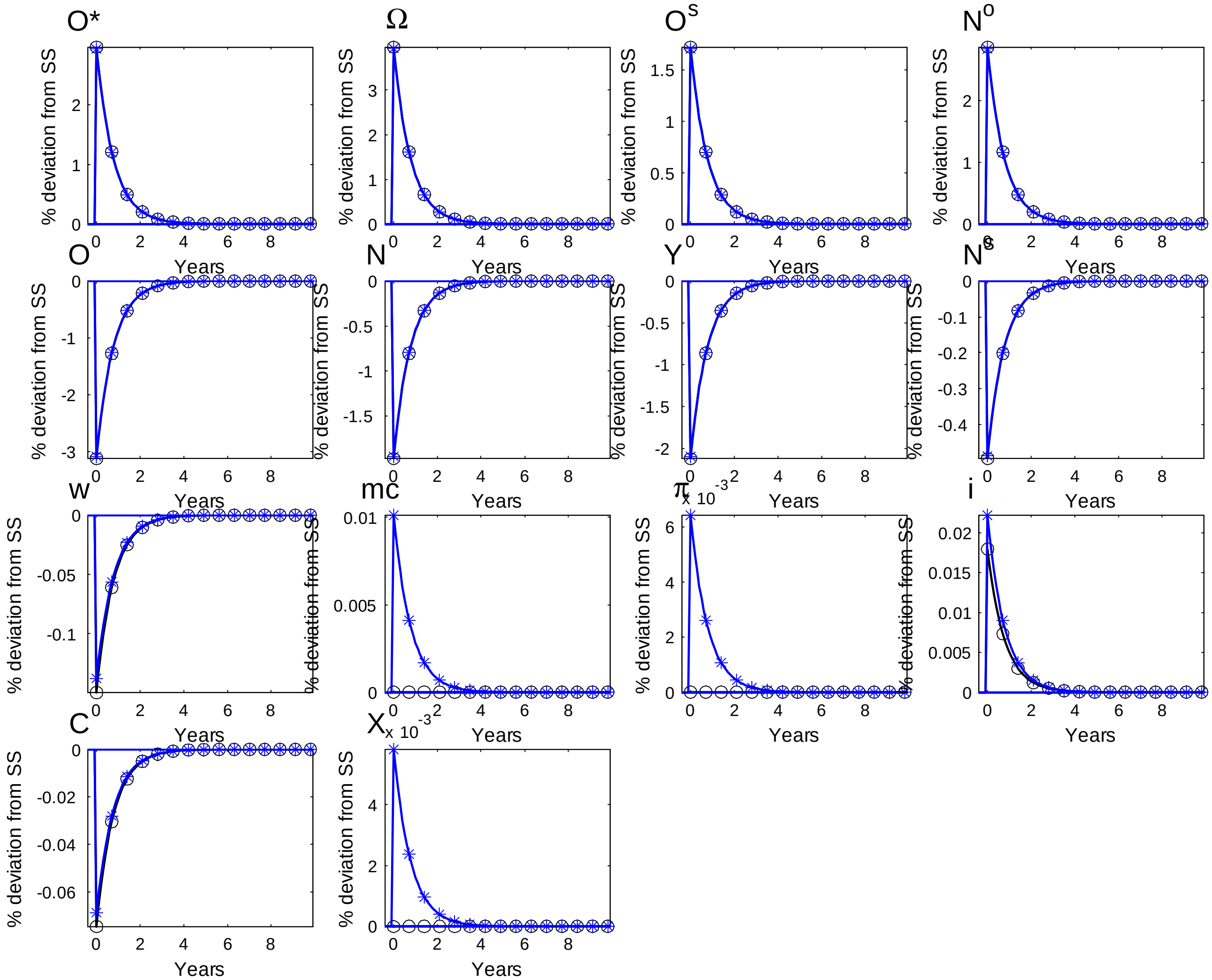

Figure 4: Responses to an oil price shock under optimal policy. Black —○—: Optimal Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}_t^n)$.

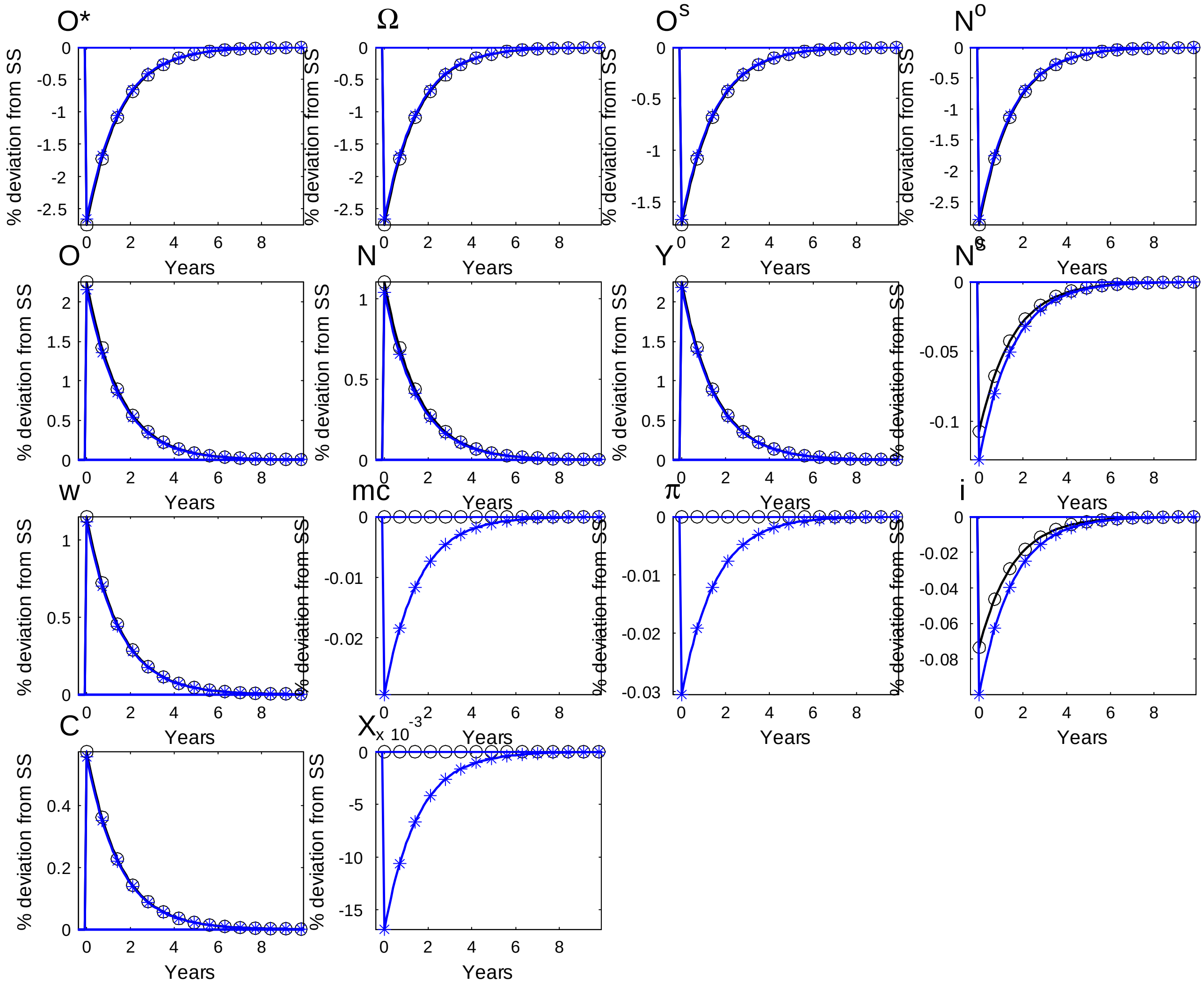

Figure 5: Responses to a productivity shock in the final goods sector under optimal policy.
Black —○—: Optimal Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}^n_t)$

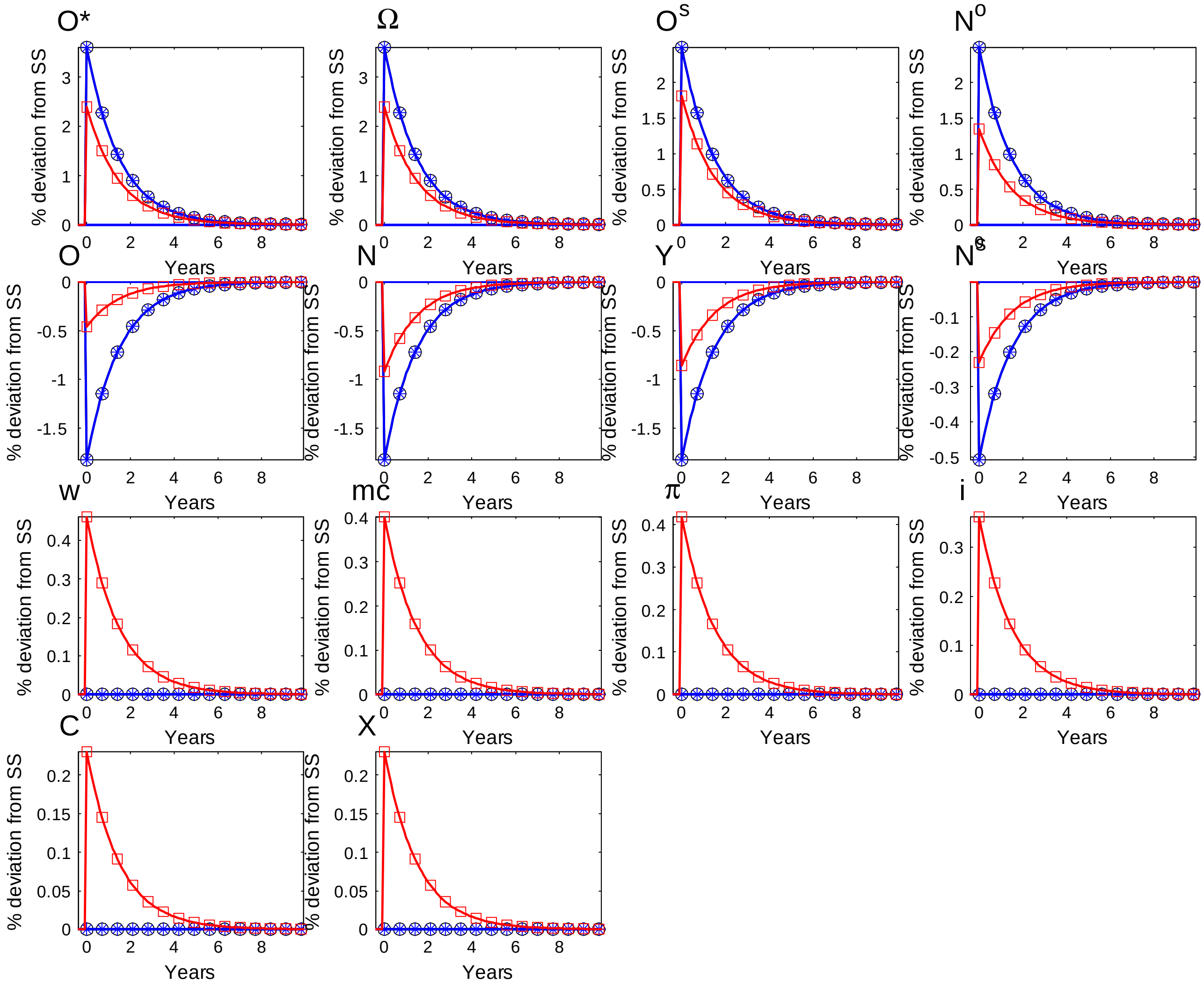

Figure 6: Responses to a productivity shock in the oil sector under optimal policy. Black —○—: Optimal Blue —*—: $i_t = 3\pi_t + .5(\hat{c}_t - \hat{c}_t^n)$ Red—□—:$i_t = 3\pi_t + 1/3(\hat{y}_t - \hat{y}_t^n)$

B Mathematical Appendix

This appendix contains the proofs of all propositions as well as the relevant mathematical derivations. I start by proving the results of the static model and proceed with the proofs and derivations of the dynamic model.

Proof of Proposition 2. Note that the results stated under Proposition 2 are not dependent on the form of the utility function. Taking the partial derivatives of (18), (19),

(20) and (21) with respect to P_t^O, I obtain

$$i)\ \frac{\partial W_t}{\partial P_t^O} = -(1-\alpha)^2 A_t^{1/\alpha}\left[\frac{(1-\alpha)}{P_t^O}\right]^{\frac{1}{\alpha}-2}\frac{1}{(P_t^O)^2} = -A_t^{1/\alpha}\left[\frac{(1-\alpha)}{P_t^O}\right]^{\frac{1}{\alpha}} = -o_t < 0.$$

$$ii)\ \frac{\partial o_t}{\partial P_t^O} = -\frac{1}{\alpha}\left[\frac{A_t(1-\alpha)}{P_t^O}\right]^{\frac{1}{\alpha}-1}\frac{A_t(1-\alpha)}{(P_t^O)^2} = -\frac{1}{\alpha}\left[\frac{A_t(1-\alpha)}{P_t^O}\right]^{\frac{1}{\alpha}}\frac{1}{P_t^O} = -\frac{o_t}{\alpha P_t^O} < 0.$$

$$iii)\ \frac{\partial N_t^o}{\partial P_t^O} = \frac{1}{1-\nu}\left[\frac{\nu Z_t}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}}\right]^{\frac{1}{1-\nu}-1}\frac{\nu Z_t}{\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_t}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}-1}$$
$$= \frac{1}{1-\nu}\left[\frac{\nu Z_t}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}}\right]^{\frac{1}{1-\nu}}\frac{1}{\alpha P_t^O} = \frac{N_t^o}{1-\nu}\frac{1}{\alpha P_t^O} > 0.$$

$$iv)\ \frac{\partial O_t^S}{\partial P_t^O} = \frac{\nu}{1-\nu}\left[\frac{\nu Z_t^{\frac{1}{\nu}}}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}}\right]^{\frac{\nu}{1-\nu}-1}\frac{\nu Z_t^{\frac{1}{\nu}}}{\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_t}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}-1}$$
$$= \frac{\nu}{1-\nu}\left[\frac{\nu Z_t^{\frac{1}{\nu}}}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\left(\frac{P_t^O}{A_t}\right)^{\frac{1}{\alpha}}\right]^{\frac{\nu}{1-\nu}}\frac{1}{\alpha P_t^O} = \frac{\nu O_t^S}{1-\nu}\frac{1}{\alpha P_t^O} > 0.$$

For $v)$ and $viii)$, I assume a general form of the utility function:

$$U_t = u(C_t, N_t^S) \tag{48}$$

We are then interested in finding the signs of $\frac{\partial N_t}{\partial P_t^O}$, $\frac{\partial N_t^S}{\partial P_t^O}$ and $\frac{\partial U_t}{\partial P_t^O}$. Differentiating (48) with respect to P_t^O I obtain:

$$\frac{\partial U_t}{\partial P_t^O} = u_c\frac{\partial C_t}{\partial P_t^O} + u_N\frac{\partial N_t^S}{\partial P_t^O}, \tag{49}$$

where $u_c \equiv \frac{\partial u}{\partial C_t}$ and $u_N \equiv \frac{\partial u}{\partial N_t^S}$. Note that (49) can be written as

$$\frac{\partial U_t}{\partial P_t^O} = u_C\frac{\partial C_t}{\partial P_t^O} - W_t u_c\frac{\partial N_t^S}{\partial P_t^O}, \tag{50}$$

where I have used the first order condition for the household ($u_N = -W_t u_C$) and the fact that $P_t = 1$. Now consider the following system of equations:

$$Y_t + P_t^O O_t^* = C_t, \tag{51}$$

$$u_N = -W_t u_C. \tag{52}$$

The first of these equations is the resource constraint. The second equation is the familiar Euler condition associated with utility function (48). Using the final good firm's first order conditions, equation (51) can be rewritten as

$$A_t N_t o_t^{1-\alpha} + P_t^O [Z_t (N_t^o)^\nu - o_t N_t] = C_t,$$
$$N_t W_t + P_t^O Z_t (N_t^o)^\nu = C_t.$$

Hence, the previous system of equations now becomes:

$$N_t W_t + P_t^O Z_t (N_t^o)^\nu = C_t, \tag{53}$$

$$u_N = -W_t u_C. \tag{54}$$

Note that, from (18), and (20), W_t and N_t^o are known and are functions of P_t^O. Therefore, for given values of u_C and u_N, this is a 2-equation system in N_t and C_t. I now proceed to compute both $\frac{\partial C_t}{\partial P_t^O}$ and $\frac{\partial N_t}{\partial P_t^O}$, which are needed to calculate the value of (49). In order to compute these derivatives, totally differentiate (53) and (54) with respect to P_t^O to obtain

$$N_t \frac{\partial W_t}{\partial P_t^O} + W_t \frac{\partial N_t}{\partial P_t^O} + P_t^O Z_t \nu (N_t^o)^{\nu-1} \frac{\partial N_t^o}{\partial P_t^O} + Z_t (N_t^o)^\nu = \frac{\partial C_t}{\partial P_t^O},$$
$$[u_{NC} + W_t u_{CC}] \frac{\partial C_t}{\partial P_t^O} + [u_{NN} + W_t u_{CN}] \frac{\partial N_t^S}{\partial P_t^O} = -u_C \frac{\partial W_t}{\partial P_t^O},$$

where $u_{NC} \equiv \frac{\partial^2 u}{\partial N_t^S \partial C_t} < 0, u_{CC} \equiv \frac{\partial^2 u}{\partial C_t^2} < 0$ and $u_{NN} \equiv \frac{\partial^2 u}{\partial N_t^2} < 0$. Using the first order condition for the oil sector, as well as the relations $\frac{\partial W_t}{\partial P_t^O} = -o_t$, $N_t o_t = O_t$ and $O_t^S - O_t = O_t^*$

to simplify these two equations, we can rewrite the system as

$$\frac{1}{W_t}\frac{\partial C_t}{\partial P_t^O} - \frac{\partial N_t^S}{\partial P_t^O} = \frac{O_t^*}{W_t}, \tag{55}$$

$$\frac{[u_{NC} + W_t u_{CC}]}{[u_{NN} + W_t u_{CN}]}\frac{\partial C_t}{\partial P_t^O} + \frac{\partial N_t^S}{\partial P_t^O} = -\frac{u_C}{[u_{NN} + W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O}. \tag{56}$$

Adding these equations and solving for $\frac{\partial C_t}{\partial P_t^O}$ I obtain

$$\frac{\partial C_t}{\partial P_t^O} = \frac{-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{O_t^*}{W_t}}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}. \tag{57}$$

Substituting (57) into (56), I get

$$\frac{\partial N_t^S}{\partial P_t^O} = -\frac{u_C}{[u_{NN} + W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} - \frac{[u_{NC} + W_t u_{CC}]}{[u_{NN} + W_t u_{CN}]}\frac{-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{O_t^*}{W_t}}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}$$

$$= \frac{-\frac{[u_{NC}+W_t u_{CC}]u_C}{[u_{NN}+W_t u_{CN}]^2}\frac{\partial W_t}{\partial P_t^O} - \frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O}\frac{1}{W_t} + \frac{[u_{NC}+W_t u_{CC}]u_C}{[u_{NN}+W_t u_{CN}]^2}\frac{\partial W_t}{\partial P_t^O} - \frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]}\frac{O_t^*}{W_t}}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}.$$

$$\frac{\partial N_t^S}{\partial P_t^O} = \frac{-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O}\frac{1}{W_t} - \frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]}\frac{O_t^*}{W_t}}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}} < 0. \tag{58}$$

Hence, $\frac{\partial N_t}{\partial P_t^O} = \frac{\partial N_t^S}{\partial P_t^O} - \frac{\partial N_t^o}{\partial P_t^O} < 0$. Finally, using these results in (50),

$$\frac{\partial U_t}{\partial P_t^O} = u_C\frac{\partial C_t}{\partial P_t^O} - W_t u_c\frac{\partial N_t^S}{\partial P_t^O}$$

$$= \frac{-\frac{u_C^2}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{u_C O_t^*}{W_t} + \frac{u_C^2}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{u_C[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]}O_t^*}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}$$

$$= \frac{\frac{u_C O_t^*}{W_t} + \frac{u_C[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} O_t^*}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}$$

$$= \frac{u_C O_t^* \left[\frac{1}{W_t} + \frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]}\right]}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}}.$$

$$\frac{\partial U_t}{\partial P_t^O} = u_C O_t^* > 0. \tag{59}$$

To prove vi), note that $\frac{\partial Y_t}{\partial P_t^O} = \frac{\partial(A_t N_t o_t^{1-\alpha})}{\partial P_t^O} = (1-\alpha) o_t^{-\alpha} A_t N_t \frac{\partial o_t}{\partial P_t^O} + o_t^{1-\alpha} A_t \frac{\partial N_t}{\partial P_t^O}$. But from ii) and v), we know that $\frac{\partial o_t}{\partial P_t^O}$ and $\frac{\partial N_t}{\partial P_t^O}$ are both negative, which implies that $\frac{\partial Y_t}{\partial P_t^O} < 0$. Now note that $\frac{\partial O_t}{\partial P_t^O} = \frac{\partial(o_t N_t)}{\partial P_t^O} = o_t \frac{\partial N_t}{\partial P_t^O} + N_t \frac{\partial o_t}{\partial P_t^O} < 0$, which also follows from ii) and v). Taking this into account, together with iv), implies that $\frac{\partial O_t^*}{\partial P_t^O} = \underset{(+)}{\frac{\partial O_t^S}{\partial P_t^O}} - \underset{(-)}{\frac{\partial O_t}{\partial P_t^O}} > 0$, which proves vii). ■

To illustrate the behavior of $\frac{\partial U_t}{\partial P_t^O}$ for different parametrizations of the utility function, I perform a numerical exercise assuming that the utility function belongs to the class of isoelastic, separable functions: $U_t = \frac{C_t^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} - \frac{\left(N_t^S\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$. In this case, (59) becomes $\frac{\partial U_t}{\partial P_t^O} = C_t^{-\frac{1}{\sigma}} O_t^*$. This quantity is evaluated numerically, that is, I solve numerically for the economy's equilibrium to find the equilibrium values of C_t and O_t^*. I do this procedure for different values of σ and φ. In particular, I assume, realistically, that $\sigma \in [0, 20]$ and $\varphi \in [0, 100]$, and I compute the value of $\frac{\partial U_t}{\partial P_t^O}$ on the rectangular grid defined by the Cartesian product $[0, 20] \times [0, 100]$. Figure 7 shows the result of this exercise. As expected, $\frac{\partial U_t}{\partial P_t^O}$ is strictly positive everywhere inside the region considered. Note that for smaller values of φ and σ, $\frac{\partial U_t}{\partial P_t^O}$ is relatively larger.

Hence, for a general class of utility functions, the conclusion is clearly that an increase in the real price of oil (P_t^O) increases welfare. As mentioned before, this result is in line with the standard theories of international trade. In this model, however, this result depends exclusively on the existence of an equilibrium with positive oil exports. The absence of oil exports would imply that an increase in P_t^O decreases welfare, since in this case the effect of

P_t^O in the economy would be purely contractionary.

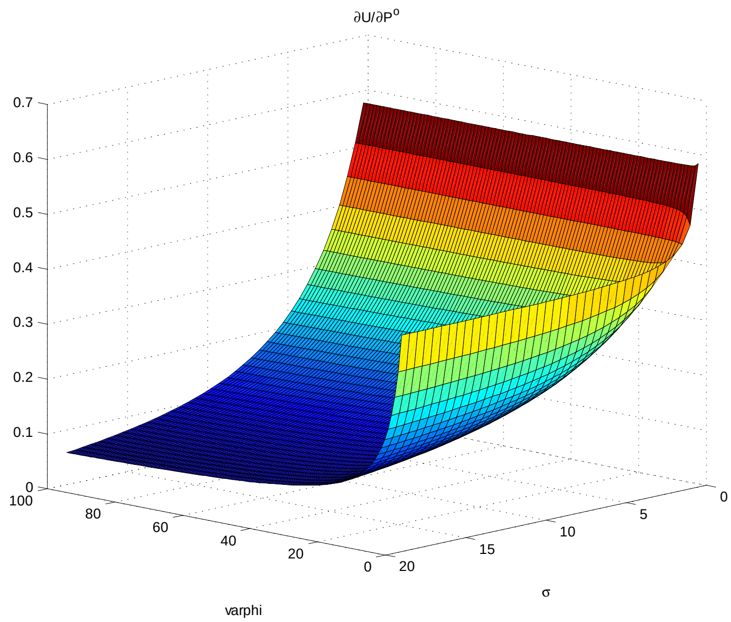

Figure 7: Increase in welfare due to an increase in the real price of oil (P_t^O) in the static model. The case of isoelastic, separable preferences.

Proof of Proposition 3. The Lagrangian for the household's maximization problem is given by (recalling that $P_t = 1$):

$$\mathcal{L} = \ln C_t - N_t^S - \lambda(C_t - W_t N_t^S - \Pi_t - \Pi_t^O).$$

The first order conditions with respect to C_t and N_t^S imply that $C_t = W_t$. This proves i). ii) follows immediately from i) and from Proposition 2. To prove iii), I first proceed to find an expression for the equilibrium level of labor in the final good sector, N_t.

Using i) and the economy's resource constraint, I obtain

$$Y_t + P_t^O O_t^* = W_t,$$

$$A_t N_t o_t^{1-\alpha} + P_t^O O_t^* = W_t,$$

$$A_t N_t o_t^{1-\alpha} + P_t^O \left[Z_t \left(N_t^o\right)^{\nu} - o_t N_t\right] = W_t,$$

$$N_t = \frac{W_t - P_t^O Z_t \left(N_t^o\right)^{\nu}}{A_t o_t^{1-\alpha} - P_t^O o_t}.$$

Using (18), (19) and (20) to substitute for W_t, o_t and N_t^o, and after some algebra, the last equation becomes

$$N_t = 1 - \frac{P_t^{O\frac{1}{\alpha(1-\nu)}} Z_t^{\frac{1}{1-\nu}} \nu^{\frac{\nu}{1-\nu}}}{\alpha^{\frac{1}{1-\nu}} (1-\alpha)^{\frac{1-\alpha}{\alpha(1-\nu)}} A_t^{\frac{1}{\alpha(1-\nu)}}}. \tag{60}$$

Note that

$$\begin{aligned}\frac{\partial N_t}{\partial P_t^o} &= -\frac{1}{\alpha(1-\nu)} \frac{P_t^{O\frac{1}{\alpha(1-\nu)}-1} Z_t^{\frac{1}{1-\nu}} \nu^{\frac{\nu}{1-\nu}}}{\alpha^{\frac{1}{1-\nu}} (1-\alpha)^{\frac{1-\alpha}{\alpha(1-\nu)}} A_t^{\frac{1}{\alpha(1-\nu)}}} \\ &= -\frac{1}{\alpha(1-\nu)} \frac{1}{P_t^O}(1-N_t) < 0.\end{aligned}$$

Now use (20), and rewrite (60) as

$$N_t = 1 - \frac{N_t^o}{\nu}. \tag{61}$$

Now, from the first order condition for the oil sector, we have

$$\nu Z_t P_t^O \left(N_t^O\right)^{\nu-1} = W_t, \tag{62}$$

$$\nu P_t^O O_t^S = W_t N_t^O.$$

Hence

$$N_t^O = \frac{\nu P_t^O O_t^S}{C_t} = \frac{\nu(P_t^O O_t + P_t^O O_t^*)}{C_t}. \tag{63}$$

However, note that from the zero profit condition of the final good sector firm, we have that $Y_t = P_t^O O_t + W_t N_t$. Using this equation to substitute for $P_t^O O_t$ in (63), we get

$$N_t^O = \frac{\nu(Y_t + P_t^O O_t^* - W_t N_t)}{C_t} = \nu - \frac{W_t N_t}{C_t}.$$

Hence, $N_t^O < \nu < 1$, which in turn implies, from (61), that $N_t < 1$. Now, from the final good firm's first order condition for labor, we have that $N_t = \frac{\alpha Y_t}{W_t} = \frac{\alpha Y_t}{C_t} < \alpha$. ■

Proof of Proposition 4. I start by proving i). Using the resource constraint, the market clearing condition for oil, and the fact that $N_t^o = 1 - N_t$, we obtain

$$\begin{aligned} C_t &= Y_t + P_t^O O_t^* \\ &= A_t N_t o_t^{1-\alpha} + P_t^O \left[Z_t \left(N_t^o \right)^\nu - o_t N_t \right] \\ &= (1 - N_t^o)(A_t o_t^{1-\alpha} - P_t^O o_t) + P_t^O Z_t \left(N_t^o \right)^\nu . \end{aligned} \tag{64}$$

Now note that we can rewrite (19) as

$$P_t^O o_t = A_t (1-\alpha) o_t^{1-\alpha}. \tag{65}$$

Using this equation in (64), we can write

$$C_t = (1 - N_t^o) \alpha A_t o_t^{1-\alpha} + P_t^O Z_t \left(N_t^o \right)^\nu .$$

Differentiating with respect to P_t^O:

$$\begin{aligned} \frac{\partial C_t}{\partial P_t^O} &= \alpha A_t \left[(1 - N_t^o)(1-\alpha) o_t^{-\alpha} \frac{\partial o_t}{\partial P_t^O} - o_t^{1-\alpha} \frac{\partial N_t^o}{\partial P_t^O} \right] + Z_t P_t^O \nu \left(N_t^o \right)^{\nu - 1} \frac{\partial N_t^o}{\partial P_t^O} + Z_t \left(N_t^o \right)^\nu \\ &= \alpha A_t (1 - N_t^o)(1-\alpha) o_t^{-\alpha} \frac{\partial o_t}{\partial P_t^O} + \frac{\partial N_t^o}{\partial P_t^O} \left[Z_t P_t^O \nu \left(N_t^o \right)^{\nu-1} - \alpha A_t o_t^{1-\alpha} \right] + Z_t \left(N_t^o \right)^\nu . \end{aligned}$$

Let us first focus on the term in square brackets.

$$H \equiv Z_t P_t^O \nu \left(N_t^o \right)^{\nu-1} - \alpha A_t o_t^{1-\alpha}.$$

Note also that we can rewrite (20) as $\left(N_t^o \right)^{\nu-1} = \frac{\alpha}{(1-\alpha)\nu Z_t} o_t$. Using this result together with equation (65), we obtain

$$
\begin{aligned}
H &= \frac{\alpha}{(1-\alpha)} P_t^O o_t - \alpha A_t o_t^{1-\alpha} \\
&= \frac{\alpha}{(1-\alpha)} A_t (1-\alpha) o_t^{1-\alpha} - \alpha A_t o_t^{1-\alpha} = 0.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\partial C_t}{\partial P_t^O} &= \alpha A_t (1 - N_t^o)(1-\alpha) o_t^{-\alpha} \frac{\partial o_t}{\partial P_t^O} + Z_t \left(N_t^o\right)^{\nu} \\
&= \alpha A_t N_t (1-\alpha) o_t^{-\alpha} \frac{\partial o_t}{\partial P_t^O} + O_t^S \\
&= \alpha A_t N_t (1-\alpha) o_t^{-\alpha} \frac{\partial o_t}{\partial P_t^O} + o_t N_t + O_t^*. \qquad (66)
\end{aligned}
$$

However, from result ii) of Proposition 2, we have that $\frac{\partial o_t}{\partial P_t^O} = -\frac{o_t}{\alpha P_t^O}$. Substituting this in (66),

$$
\begin{aligned}
\frac{\partial C_t}{\partial P_t^O} &= -A_t N_t (1-\alpha) o_t^{1-\alpha} \frac{1}{P_t^O} + o_t N_t + O_t^* \\
&= N_t \left(-\frac{A_t (1-\alpha) o_t^{1-\alpha}}{P_t^O} + o_t \right) + O_t^*.
\end{aligned}
$$

But, by (19), this becomes

$$
\begin{aligned}
\frac{\partial C_t}{\partial P_t^O} &= N_t \left(-o_t^{\alpha} o_t^{1-\alpha} + o_t \right) + O_t^* \\
&= O_t^* > 0.
\end{aligned}
$$

To show that property ii) must hold, we first notice that, from the market clearing condition for labor, we must have $N_t = 1 - N_t^o$. Hence, in this economy, in order for the equilibrium level of labor in the final good sector to be positive, it must be true that $N_t^o < 1$.

Now, from the first order condition for the oil sector, equation (62), we have that

$$P_t^O = \frac{W_t \left(N_t^O\right)^{1-\nu}}{\nu Z_t} < \frac{W_t}{\nu Z_t}.$$

Hence, we have found an upper bound for the real price of oil. This condition is necessary in order to have a positive value for the equilibrium level of labor in the final good sector, N_t. ■

Proof of Proposition 5. In this case the resource constraint becomes

$$C_t = Y_t + \Omega_t - \bar{\tau}\Pi_t^O.$$

Using (12), we can rewrite this equation as

$$\begin{aligned} C_t &= Y_t + \Omega_t - \bar{\tau}\nu^{\frac{\nu}{1-\nu}} (1-\nu) \frac{\left(Z_t P_t^O\right)^{\frac{1}{1-\nu}}}{W_t^{\frac{\nu}{1-\nu}}} \\ &= Y_t + \Omega_t - \bar{\tau}\bar{\Pi}_t \left(P_t^O\right)^{\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}} \\ &= N_t W_t + P_t^O Z_t \left(N_t^o\right)^{\nu} - \bar{\tau}\bar{\Pi}_t \left(P_t^O\right)^{\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}}, \end{aligned}$$

where $\bar{\Pi}_t \equiv \frac{\nu^{\frac{\nu}{\nu-1}}(1-\nu) Z_t^{\frac{1}{1-\nu}}}{\alpha^{\frac{\nu}{\nu-1}}(1-\alpha)^{\frac{1-a}{\alpha}\frac{\nu}{\nu-1}} A_t^{\frac{\nu}{\alpha(1-\nu)}}}$. Now consider the following pair of equations:

$$N_t W_t + P_t^O Z_t \left(N_t^o\right)^{\nu} - \bar{\tau}\bar{\Pi}_t \left(P_t^O\right)^{\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}} = C_t, \tag{67}$$

$$u_N = -W_t u_C. \tag{68}$$

I now follow the same procedure described under the proof of Proposition 2. Differentiation of the previous two equations with respect to P_t^O yields

$$\begin{aligned} N_t \frac{\partial W_t}{\partial P_t^O} + W_t \frac{\partial N_t}{\partial P_t^O} + P_t^O Z_t \nu \left(N_t^o\right)^{\nu-1} \frac{\partial N_t^o}{\partial P_t^O} + Z_t \left(N_t^o\right)^{\nu} - \bar{\tau}\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}\bar{\Pi}_t \left(P_t^O\right)^{\frac{\nu}{\alpha(1-\nu)}} &= \frac{\partial C_t}{\partial P_t^O}, \\ \left[u_{NC} + W_t u_{CC}\right] \frac{\partial C_t}{\partial P_t^O} + \left[u_{NN} + W_t u_{CN}\right] \frac{\partial N_t^S}{\partial P_t^O} &= -u_C \frac{\partial W_t}{\partial P_t^O}. \end{aligned}$$

Using equilibrium conditions to simplify,

$$\frac{1}{W_t}\frac{\partial C_t}{\partial P_t^O} - \frac{\partial N_t^S}{\partial P_t^O} = \frac{O_t^*}{W_t} - \bar{\tau}\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}\bar{\Pi}_t\frac{\left(P_t^O\right)^{\frac{\nu}{\alpha(1-\nu)}}}{W_t},$$
$$\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]}\frac{\partial C_t}{\partial P_t^O} + \frac{\partial N_t^S}{\partial P_t^O} = -\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O}.$$

Adding these equations and solving for $\frac{\partial C_t}{\partial P_t^O}$, I obtain

$$\frac{\partial C_t}{\partial P_t^O} = \frac{-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{O_t^*}{W_t} - \bar{\tau}\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}\bar{\Pi}_t\frac{\left(P_t^O\right)^{\frac{\nu}{\alpha(1-\nu)}}}{W_t}}{\frac{[u_{NC}+W_t u_{CC}]}{[u_{NN}+W_t u_{CN}]} + \frac{1}{W_t}},$$

which is positive if

$$-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{O_t^*}{W_t} - \bar{\tau}\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}\bar{\Pi}_t\frac{\left(P_t^O\right)^{\frac{\nu}{\alpha(1-\nu)}}}{W_t} > 0.$$

By defining

$$\tau' \equiv \frac{-\frac{u_C}{[u_{NN}+W_t u_{CN}]}\frac{\partial W_t}{\partial P_t^O} + \frac{O_t^*}{W_t}}{\frac{\alpha+(1-\alpha)\nu}{\alpha(1-\nu)}\bar{\Pi}_t\frac{\left(P_t^O\right)^{\frac{\nu}{\alpha(1-\nu)}}}{W_t}}$$

we conclude that if $\bar{\tau} \lesseqgtr \tau'$ then $\frac{\partial C_t}{\partial P_t^O} \gtreqless 0$. ■

Proof of Proposition 6. Loglinearizing (17), (33) and (39) around the deterministic steady state, I obtain

$$\hat{n}_t^s = \mu\hat{n}_t + (1-\mu)\hat{n}_t^o, \tag{69}$$
$$\hat{w}_t = \frac{1}{\varphi}\hat{n}_t^s + \frac{1}{\sigma}\hat{c}_t, \tag{70}$$
$$\widehat{mc}_t = \alpha\hat{w}_t + (1-\alpha)\hat{q}_t - \hat{a}_t, \tag{71}$$

where $\mu \equiv \frac{N}{N^S}$ and $\hat{w}_t \equiv \ln(\frac{W_t}{P_t}) - \ln(\frac{W}{P})$. Now, loglinearization of (11), yields

$$\hat{n}_t^o = \frac{1}{\nu-1}\hat{w}_t - \frac{1}{\nu-1}\hat{q}_t - \frac{1}{\nu-1}\hat{z}_t. \tag{72}$$

Since labor demand for the final good firm is given by

$$N_t(j) = \frac{MC_t \alpha Y(j)}{W_t}, \tag{73}$$

we have that

$$N_t = \frac{\Upsilon \frac{W_t^{\alpha}\left(P_t^O\right)^{1-\alpha}}{A_t} \alpha \int_0^1 Y_t(j)dj}{W_t} = \alpha \Upsilon \frac{W_t^{\alpha-1}\left(P_t^O\right)^{1-\alpha}}{A_t} Y_t, \tag{74}$$

where $\Upsilon \equiv \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$. Loglinearizing, I obtain

$$\hat{n}_t = \hat{y}_t - \hat{a}_t + (1-\alpha)\hat{q}_t - (1-\alpha)\hat{w}_t. \tag{75}$$

Solving for $\hat{w}_t$ in (71) and substituting the resulting relation in (72) and (75), I obtain, after simplifying,

$$\begin{aligned}
\hat{n}_t^o &= \frac{1}{(\nu-1)\,\alpha}\widehat{mc}_t - \frac{1}{(\nu-1)\,\alpha}\hat{q}_t + \frac{1}{(\nu-1)\,\alpha}\hat{a}_t - \frac{1}{(\nu-1)}\hat{z}_t, \\
\hat{n}_t &= \hat{y}_t - \frac{1}{\alpha}\hat{a}_t + \frac{(1-\alpha)}{\alpha}\hat{q}_t - \frac{(1-\alpha)}{\alpha}\widehat{mc}_t.
\end{aligned}$$

Substituting these relations in (69) and using the resulting equation, as well as (71), in equation (70) yields, after some algebra

$$\begin{aligned}
&\frac{1}{\varphi}\mu\hat{y}_t + \left\{\frac{1}{\varphi\alpha}\left[-\mu + \frac{1-\mu}{\nu-1}\right] - \frac{1}{\alpha}\right\}\hat{a}_t + \left\{\frac{1}{\varphi\alpha}\left[\mu(1-\alpha) - \frac{1-\mu}{\nu-1}\right] + \frac{1-\alpha}{\alpha}\right\}\hat{q}_t \\
&\quad -\frac{1}{\varphi}\frac{1-\mu}{\nu-1}\hat{z}_t + \frac{1}{\sigma}\hat{c}_t = \left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu - \frac{1-\mu}{\nu-1}\right]\right\}\widehat{mc}_t.
\end{aligned} \tag{76}$$

Loglinearizing the resource constraint results in: $\hat{c}_t = \eta y_t + (1-\eta)\hat{\Omega}_t$, where $\eta \equiv \frac{Y}{C}$. Using this equation to substitute for $\hat{c}_t$ in (76), rearranging, and solving for $\widehat{mc}_t$, I obtain

$$\widehat{mc}_t = k_1\hat{y}_t - k_2\hat{a}_t + k_3\hat{q}_t + k_4\hat{z}_t + k_5\hat{\Omega}_t, \tag{77}$$

where

$$k_1 \equiv \frac{\left(\frac{\mu}{\varphi} + \frac{\eta}{\sigma}\right)}{\left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu + \frac{1-\mu}{1-\nu}\right]\right\}} > 0,$$

$$k_2 \equiv \frac{\frac{1}{\varphi\alpha}\left[\left(\frac{1-\mu\nu}{1-\nu}\right)\right] + \frac{1}{\alpha}}{\left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu + \frac{1-\mu}{1-\nu}\right]\right\}} > 0,$$

$$k_3 \equiv \frac{\frac{1}{\varphi\alpha}\left[\mu(1-\alpha) + \frac{1-\mu}{1-\nu}\right] + \frac{1-\alpha}{\alpha}}{\left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu + \frac{1-\mu}{1-\nu}\right]\right\}} > 0,$$

$$k_4 \equiv \frac{\frac{1}{\varphi}\frac{1-\mu}{1-\nu}}{\left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu + \frac{1-\mu}{1-\nu}\right]\right\}} > 0,$$

$$k_5 \equiv \frac{\frac{1-\eta}{\sigma}}{\left\{\frac{1}{\alpha} + \frac{1}{\varphi\alpha}\left[(1-\alpha)\mu + \frac{1-\mu}{1-\nu}\right]\right\}} > 0.$$

Finally, using (77) to substitute for $\widehat{mc}_t$ in (43) yields the desired result. ■

Proof of Proposition 7. I now proceed to derive a second order approximation to the household's utility function. First, write (22), the per-period utility function, as

$$U_t = u(C_t) - \upsilon(N_t^S) \tag{78}$$

Writing $u(C_t)$ as a second order Taylor expansion around the steady state, I obtain

$$u(C_t) = u(C) + u_c(C)(C_t - C) + \frac{1}{2}u_{cc}(C)(C_t - C)^2 + O(\|\xi\|)^3, \tag{79}$$

where $O(\|\xi\|)^3$ encloses all terms of third and higher order. Now, note that $C_t = Ce^{\hat{c}_t} = C\left[1 + \hat{c}_t + \frac{1}{2}\hat{c}_t^2 + O(\|\xi\|)^3\right]$. Using this relationship to substitute for $C_t - C$ in (79), yields

$$u(C_t) = u_c(C)C\left(\hat{c}_t + \frac{1-\frac{1}{\sigma}}{2}\hat{c}_t^2\right) + t.i.p. + O(\|\xi\|)^3, \tag{80}$$

where $t.i.p.$ captures the terms that are independent of policy.

Similarly, a second order Taylor expansion of $\upsilon(N_t^S)$ with $\varphi = \infty$ results in

$$\begin{aligned}\upsilon(N_t^S) &= \upsilon_{N^S}(N^S)N^S\left[\hat{n}_t^S + \frac{1}{2}(\hat{n}_t^S)^2\right] + t.i.p. + O(\|\xi\|)^3 \\ &= \upsilon_{N^S}(N^S)N^S\left\{(1-\gamma)\hat{n}_t + \gamma\hat{n}_t^o + \frac{1}{2}\left[(1-\gamma)\hat{n}_t^2 + \gamma(\hat{n}_t^o)^2\right]\right\} \\ &\quad + t.i.p. + O(\|\xi\|)^3,\end{aligned} \tag{81}$$

where $(1-\gamma) \equiv \frac{N}{N^S}$. It is important to mention at this point that, in order to obtain the correct second order expansion for $\upsilon(N_t^S)$, one needs to take into account the second order expansion of $\hat{n}_t$. This is true since otherwise one would be ignoring second order terms in $\hat{c}_t$ that enter (81) through $\hat{n}_t$. To see this, note that

$$\begin{aligned}N_t &= \alpha\Upsilon\frac{W_t^{\alpha-1}\left(P_t^O\right)^{1-\alpha}}{A_t}\int_0^1 Y_t(j)dj \\ &= \alpha\Upsilon\frac{W_t^{\alpha-1}\left(P_t^O\right)^{1-\alpha}}{A_t}(C_t - Q_tO_t^*)\int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\theta}dj,\end{aligned} \tag{82}$$

so that a first order loglinear approximation to the above equation would be ignoring second order terms that contribute to the second order loglinear approximation of $\upsilon(N_t^S)$. Therefore, the next step is to find a second order Taylor expansion for N_t in terms of $\hat{c}_t$ and the price dispersion measure $\Delta_t \equiv \ln[\Lambda_t]$, where $\Lambda_t \equiv \int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\theta}dj$. From the first order condition for oil for a final good producing firm, we have that

$$O_t = (1-\alpha)\Upsilon\frac{\left(\frac{W_t}{P_t}\right)^{\alpha}\left(\frac{P_t^O}{P_t}\right)^{-\alpha}}{A_t}(C_t - Q_tO_t^*)\Lambda_t.$$

Now consider the market clearing condition for labor

$$\begin{aligned}O_t^S &= O_t + O_t^*, \\ Z_t\left(N_t^o\right)^{\nu} &= (1-\alpha)\Upsilon\frac{\left(\frac{W_t}{P_t}\right)^{\alpha}\left(\frac{P_t^O}{P_t}\right)^{-\alpha}}{A_t}(C_t - Q_tO_t^*)\Lambda_t + O_t^*.\end{aligned}$$

Substituting for N_t^o from the oil sector's first order condition, this equation becomes

$$\left(\frac{\nu Z_t^{1/\nu}\frac{P_t^O}{P_t}}{\frac{W_t}{P_t}}\right)^{\frac{\nu}{1-\nu}} = (1-\alpha)\Upsilon\frac{\left(\frac{W_t}{P_t}\right)^{\alpha}\left(\frac{P_t^O}{P_t}\right)^{-\alpha}}{A_t}(C_t - Q_tO_t^*)\Lambda_t + O_t^*.$$

Using the fact that $\frac{W_t}{P_t} = C_t^{\frac{1}{\sigma}}$ and solving for O_t^* results in

$$O_t^* = \frac{\nu^{\frac{\nu}{1-\nu}} Z_t^{\frac{1}{1-\nu}} Q_t^{\frac{\nu}{1-\nu}} C_t^{\frac{-\nu}{\sigma(1-\nu)}} - (1-\alpha)\Upsilon C_t^{\frac{\alpha}{\sigma}+1} Q_t^{-\alpha}\Lambda_t A_t^{-1}}{1-(1-\alpha)\Upsilon C_t^{\frac{\alpha}{\sigma}} Q_t^{1-\alpha}\Lambda_t A_t^{-1}}.$$

Using this equation to substitute for O_t^* in (82) yields

$$N_t = \frac{\alpha C_t^{1-\frac{1}{\sigma}} - \alpha\nu^{\frac{\nu}{1-\nu}} Z_t^{\frac{1}{1-\nu}} Q_t^{\frac{1}{1-\nu}} C_t^{\frac{-1}{\sigma(1-\nu)}}}{A_t\Upsilon^{-1}C_t^{-\frac{\alpha}{\sigma}} Q_t^{\alpha-1}\Lambda_t^{-1} - (1-\alpha)}. \tag{83}$$

A second order expansion in log-deviations of this equation can be written as

$$\begin{aligned}\hat{n}_t =& n_q\hat{q}_t + n_c\hat{c}_t + n_a\hat{a}_t + n_z\hat{z}_t + n_\Delta\Delta_t + \frac{1}{2}n_{c^2}\hat{c}_t^2 \\ &+ n_{cq}\hat{c}_t\hat{q}_t + n_{ca}\hat{c}_t\hat{a}_t + n_{cz}\hat{c}_t\hat{z}_t + O(\|\xi\|)^3,\end{aligned} \tag{84}$$

where the coefficients in front of the variables depend on steady state relations. Using (84) and (72) in (81), and substituting the resulting equation, together with (80), in (78), I obtain the second order approximation to the per-period utility function:

$$\begin{aligned}U_t =& -\frac{1}{2}u_c(C)C\left\{D_1\hat{c}_t + D_2\Delta_t + D_3\hat{c}_t^2 + D_4\hat{c}_t\hat{q}_t + D_5\hat{c}_t\hat{a}_t + D_6\hat{c}_t\hat{z}_t\right\} \\ &+ t.i.p. + O(\|\xi\|)^3,\end{aligned} \tag{85}$$

where

$$D_1 = -2\left\{1 - \Psi\left[(1-\gamma)n_c - \frac{\gamma}{1-\nu}\frac{1}{\sigma}\right]\right\},$$
$$D_2 = 2\Psi(1-\gamma)n_\Delta,$$
$$D_3 = \left\{(\frac{1}{\sigma}-1) + 2\Psi\left[\frac{(1-\gamma)}{2}\left(n_{c^2} + (n_c)^2\right) + \frac{1}{2}\gamma\left(\frac{1}{1-\nu}\frac{1}{\sigma}\right)^2\right]\right\},$$
$$D_4 = 2\Psi\left\{(1-\gamma)\left(n_{cq} + n_c n_q\right) - \gamma\left(\frac{1}{1-\nu}\right)^2\frac{1}{\sigma}\right\},$$
$$D_5 = 2\Psi\left\{(1-\gamma)\left(n_{ca} + n_c n_a\right)\right\},$$
$$D_6 = 2\Psi\left\{(1-\gamma)\left(n_{cz} + n_c n_z\right) - \gamma\left(\frac{1}{1-\nu}\right)^2\frac{1}{\sigma}\right\},$$
$$\text{and } \Psi \equiv \frac{\upsilon_{N^S}(N^S)N^S}{u_c(C)C}.$$

I now assume that the production subsidy exactly offsets the markup distortion, so that $\frac{\theta}{(\theta-1)(1+\zeta)} = 1$. In this case, D_1 becomes identically 0. To see this, recall that in steady state, $P = \frac{\theta}{(\theta-1)(1+\zeta)}MC = MC$. I also assume that $A = Z = 1$ in the steady state. Then, we must show that

$$1 - \Psi\left[(1-\gamma)n_c - \frac{\gamma}{1-\nu}\frac{1}{\sigma}\right] = 0,$$

or, equivalently

$$1 - \frac{W}{P}\frac{N^S}{C}\left\{\frac{N}{N^S}\left[\frac{C}{Y} + \frac{\nu}{1-\nu}\frac{1}{\sigma}\frac{C}{Y} - \frac{\alpha}{\sigma}\frac{\nu}{1-\nu}\right]\frac{1}{\alpha} - \frac{N^O}{N^S}\frac{1}{1-\nu}\frac{1}{\sigma}\right\} = 0,$$

where I have substituted for the corresponding value of n_c that results from the Taylor expansion of (83), and have used the household's first order condition for labor supply to substitute for $\frac{\upsilon_{N^S}(N^S)}{u_c(C)}$. Now, recalling that in steady state $\frac{C}{Y} = \frac{1}{1-\chi}$, the left hand side of the previous equation can be rewritten as

$$1 - \frac{W}{P}\frac{N}{C}\left[\frac{1}{1-\chi} + \frac{\nu}{1-\nu}\frac{1}{\sigma}\frac{1}{1-\chi} - \frac{\alpha}{\sigma}\frac{\nu}{1-\nu}\right]\frac{1}{\alpha} + \frac{W}{P}\frac{N^O}{C}\frac{1}{1-\nu}\frac{1}{\sigma}. \tag{86}$$

With this in mind, note that

$$\begin{aligned}\frac{W}{P} &= \frac{W}{MC} \\ &= \frac{W^{1-\alpha}\left(P^{O}\right)^{\alpha-1}}{\Upsilon}.\end{aligned} \tag{87}$$

But, from the first order condition of the oil sector, we have that

$$\frac{W}{P} = \frac{\nu Q}{(O^{S})^{\frac{1-\nu}{\nu}}}. \tag{88}$$

Using (88) in (87), it becomes

$$\frac{W}{P} = \left(\frac{\nu}{(O^{S})^{\frac{1-\nu}{\nu}}}\right)^{1-\alpha} \Upsilon^{-1}.$$

Then,

$$\begin{aligned}\frac{W}{P}\frac{N}{C} &= \left(\frac{\nu}{(O^{S})^{\frac{1-\nu}{\nu}}}\right)^{1-\alpha} \Upsilon^{-1}\left\{\alpha\Upsilon\frac{Y}{C}\left[\frac{\nu}{(O^{S})^{\frac{1-\nu}{\nu}}}\right]^{\alpha-1}\right\} \\ &= \alpha\frac{Y}{C} \\ &= \alpha(1-\chi).\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{W}{P}\frac{N^{O}}{C} &= \frac{\nu Q}{(O^{S})^{\frac{1-\nu}{\nu}}}\frac{\left(O^{S}\right)^{\frac{1}{\nu}}}{C} \\ &= \nu\frac{QO^{S}}{C} \\ &= \nu\left[1-\alpha(1-\chi)\right],\end{aligned}$$

where the third line obtains from the fact that $C - QO^{S} = \alpha Y$. Using these results in (86), yields

$$\begin{aligned}
&1-\alpha(1-\chi)\left[\frac{1}{1-\chi}+\frac{\nu}{1-\nu}\frac{1}{\sigma}\frac{1}{1-\chi}-\frac{\alpha}{\sigma}\frac{\nu}{1-\nu}\right]\frac{1}{\alpha}+\nu\left[1-\alpha(1-\chi)\right]\frac{1}{1-\nu}\frac{1}{\sigma}\\
=&1-1-\frac{\nu}{1-\nu}\frac{1}{\sigma}+\frac{\alpha}{\sigma}\frac{\nu}{1-\nu}(1-\chi)+\frac{\nu}{1-\nu}\frac{1}{\sigma}-\alpha(1-\chi)\frac{\nu}{1-\nu}\frac{1}{\sigma}=0.
\end{aligned}$$

Hence, (85) now becomes

$$U_t=-\frac{1}{2}u_c(C)C\left\{D_2\Delta_t+D_3\hat{c}_t^2+D_4\hat{c}_t\hat{q}_t+D_5\hat{c}_t\hat{a}_t+D_6\hat{c}_t\hat{z}_t\right\}+t.i.p.+O(\|\xi\|)^3.$$

Following Woodford (2003, chapter 6), we have that $\Delta_t=\frac{\theta}{2}var_j\left[\log(P_t(j)\right]$ and also that $\sum_{t=0}^{\infty}\beta^t var_j\left[\log(P_t(j)\right]=\frac{1}{\kappa}\sum_{t=0}^{\infty}\beta^t\pi_t^2+t.i.p.+O(\|\xi\|)^3$. Consolidating these results, (27) becomes

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^tU_t=\left(-\frac{1}{2}u_c(C)C\right)\mathbb{E}_0\sum_{t=0}^{\infty}\beta^tL_t+t.i.p.+O(\|\xi\|)^3,$$

where

$$L_t=\frac{D_2\theta}{2\kappa}\pi_t^2+D_3\left[\hat{c}_t-\left(\frac{-D_4\hat{q}_t-D_5\hat{a}_t-D_6\hat{z}_t}{2D_3}\right)\right]^2.$$

By defining $\lambda_1\equiv\frac{D_2\theta}{2\kappa}$, $\lambda_2\equiv D_3$, $\hat{c}_t^n\equiv\left(\frac{-D_4\hat{q}_t-D_5\hat{a}_t-D_6\hat{z}_t}{2D_3}\right)$ and $\hat{x}_t\equiv\hat{c}_t-\hat{c}_t^n$, the period loss function becomes

$$L_t=\lambda_1\pi_t^2+\lambda_2\hat{x}_t^2.$$

■