Appendix

A A Formal Model of Input Price Variation

This appendix provides a formal economic model that rationalizes the use of a flexible polynomial in output price, market share and product dummies to control for input prices, and hence the B_{ft} term in equation (18), along the lines discussed in Section 4.3.2. The model is a more general version of the models considered in Kremer (1993) and Verhoogen (2008).

We proceed in the following steps. We first show that under the assumptions of the model, the quality of *every* input is an increasing function of output quality. Next, we show that this implies that the price of *every* input will be an increasing function of output quality. In the final step, we show that output quality can be expressed as a flexible function of output price, market share and a set of product dummies. Having established a monotone relationship between input prices and output quality, this implies that the price of every input can also be expressed as a function of the above variables.

A.1 Production Function for Output Quality

In order to proceed, we must specify the production function for quality. Let v_j indicate quality of product j and ψ_i indicate the quality of input i used to produce product j.[49] The production function for output quality is given by:

$$v_j = \prod_{i=1}^{n} [\psi_i]^{\kappa_i} \cdot \omega_j \qquad with \qquad \sum \kappa_i < 1 \tag{A.1}$$

For example, with three inputs, the above production function takes the form:

$$v_j = \psi_K^{\kappa_K} \cdot \psi_L^{\kappa_L} \cdot \psi_M^{\kappa_M} \cdot \omega_j$$

This function belongs to the class of 'O-Ring' production functions discussed in Kremer (1993) and Verhoogen (2008). The particular (multiplicative) functional form is not important; the important feature is that $\frac{\partial v_j}{\partial \psi_i \partial \psi_k} > 0, \forall i, k$ and $i \neq k$. This cross-derivative implies complementarity in the quality of inputs. A direct consequence is that higher output quality requires high quality of *all* inputs (e.g., high quality material inputs are used by high-skill workers operating high-end machinery).

In addition to the production function for quality, we assume that higher quality inputs are associated with higher input prices. Let $\overline{W}_i$ denote the sectoral average of the price of input i (e.g., sectoral wage) and $W_i(\psi_i)$ the price of a specific quality ψ of input i. Then,

$$W_i(\psi_i) - \overline{W}_i = z_i \cdot \psi_i \quad \text{and} \quad z_i > 0. \tag{A.2}$$

The equation above says that in order to use higher quality inputs, a firm needs to pay higher input prices. There are many ways to justify this relationship. For example, if input markets are competitive but

[49] Here, the subscript j denotes both firm and product.

have vertical differentiation, firms must pay higher prices for higher quality inputs. So while high quality inputs are expensive, all firms pay the same input prices *conditional* on input quality.

A.2 Demand

We close the model by specifying the demand and firms' behavioral assumptions.

The indirect utility V_{nj} that consumer n derives from consuming one unit of product j can be written in general form as:

$$V_{nj} = \theta_n v_j - \alpha p_j + \varepsilon_{nj} \tag{A.3}$$

where p_j is output price, θ_n denotes the willingness to pay for quality and ε_{nj} denotes an idiosyncratic preference shock. This specification is general and encompasses all demand models commonly used in the literature. In its most general formulation, the specification above corresponds to the random coefficients model. In models of pure vertical differentiation, the utility will be given by the above expression with $\varepsilon_{nj} = 0$. A simple logit sets $\theta_n = \theta = 1$ (i.e., no observable consumer heterogeneity) and ε_{nj} is assumed to follow the extreme value distribution. In the nested logit, $\theta_n = \theta = 1$ and ε_{nj} follows the generalized extreme value distribution. Following the Industrial Organization literature, it is convenient to define the mean utility δ_j of product j as $\delta_j = v_j - \alpha p_j$. The output quality v_j is typically modeled as a function of product characteristics.

We now show how to control for quality variation across firms using observable characteristics using the specification in (A.3). Berry (1994) shows that the actual market share of a product (ms_j) is a function of product characteristics and output price:

$$ms_j = s_j(\boldsymbol{\delta}, \boldsymbol{\sigma}) = s_j(\mathbf{v}, \mathbf{p}, \boldsymbol{\vartheta}) \tag{A.4}$$

where $\boldsymbol{\sigma}$ denotes a vector of density parameters of consumer characteristics and $\boldsymbol{\vartheta}$ denotes a parameter vector. While the exact functional form is determined by choice of a particular demand structure, the general insight is that market shares are a function of product characteristics (i.e., quality) and prices. Berry (1994) shows that equation (A.4) can be inverted to obtain the mean utilities $\boldsymbol{\delta}$ as a function of the observed market shares and the density parameters to be estimated.[50] With the $\boldsymbol{\delta}'s$ in hand, quality is function of output price and the mean utility. This insight is exploited by Khandelwal (2010) who uses a nested logit model to express quality as a function of output price and conditional and unconditional market shares. In a simple logit model, quality is a function of only output prices and unconditional market shares. Here, we use a general formulation that specifies quality as a function of output price, market share and a set of product dummies:

$$v_j = v(p_j, ms_j, I) \tag{A.5}$$

The product dummies are used in lieu of product characteristics (which are not available in our data) and

[50] In the random coefficients model, the δ's are solved numerically. In simpler models, one can solve for the parameters analytically.

can accommodate more general demand specifications such as the nested logit and random coefficients model.

A.3 The Firm's Maximization Problem

Without loss of generality, we assume that firms use prices and quality as strategic variables to maximize profits. Conditional on exogenous (to the firm) input prices that are determined in competitive input markets, firms choose input qualities. These choices determine the output quality according to the quality production function in (A.1). Let mc_j denote the marginal cost of producing a product j of quality v_j. The marginal cost can be written as a function of quantity produced q_j, quality v_j, a parameter vector $\boldsymbol{\gamma}$ and productivity ω_j: $mc_j(q_j, v_j, \boldsymbol{\gamma}, \omega_j)$.

The profit function for a firm producing product j is:

$$\pi_j = N \cdot s_j \cdot [p - mc_j(q_j, v_j(\boldsymbol{\psi}, \omega_j), \boldsymbol{\gamma}, \omega_j)] \tag{A.6}$$

where N denotes the market size (number of potential consumers). Output quality v_j is now explicitly written as a function of a vector of input qualities $\boldsymbol{\psi}$ and productivity ω_j using the production function for quality in (A.1).

The first order condition with respect to price is

$$p_j = mc_j(q_j, v_j, \boldsymbol{\gamma}, \omega_j) + \frac{s_j}{|\partial s_j / \partial p_j|}. \tag{A.7}$$

The term $s_j / |\partial s_j / \partial p_j|$ represents the markup, and as shown in Berry (1994, p. 254) it equals $\frac{1}{\alpha}[s_j/(\partial s_j/\partial \delta_j)]$.

The first order condition with respect to the quality of each input i, ψ_i, is:

$$(p_j - mc_j) \cdot \frac{\partial s_j}{\partial \psi_i} - s_j \cdot \frac{\partial mc_j}{\partial \psi_i} = 0 \tag{A.8}$$

From the first order condition with respect to price, we have

$$(p_j - mc_j) = \frac{s_j}{|\partial s_j / \partial p_j|} = \frac{1}{\alpha} \cdot \frac{s_j}{\partial s_j / \partial \delta_j}. \tag{A.9}$$

Substituting this latter expression for the markup into the first order condition for input quality, we obtain:

$$s_j \cdot \frac{1}{\alpha} \cdot [1/(\partial s_j/\partial \delta_j)] \frac{\partial s_j}{\partial \psi_i} - s_j \cdot \frac{\partial mc_j}{\partial \psi_i} = 0 \tag{A.10}$$

or

$$\frac{1}{\alpha} \cdot [1/(\partial s_j/\partial \delta_j)] \left[\frac{\partial s_j}{\partial v_j} \frac{\partial v_j}{\partial \psi_i} \right] = \frac{\partial mc_j}{\partial \psi_i} \tag{A.11}$$

From $\delta_j = v_j - \alpha p_j$ follows that $\frac{\partial s_j}{\partial v_j} = \frac{\partial s_j}{\partial \delta_j}$, and the above first order condition simplifies to:

$$\frac{1}{\alpha} \cdot \frac{\partial v_j}{\partial \psi_i} = \frac{\partial mc_j}{\partial \psi_i} \tag{A.12}$$

Using the production function for quality to obtain the derivative $\frac{\partial v_j}{\partial \psi_i}$ and substituting into (A.12), we obtain

$$\psi_i = \frac{1}{\alpha} \cdot \kappa_i \cdot v_j \cdot \left[1/\frac{\partial mc_j}{\partial \psi_i}\right] \quad \forall i \tag{A.13}$$

This expression is similar to the one derived in Verhoogen (2008), but with two differences. First, as we have shown above, the above expression can be derived from a very general demand system and market structure. Second, we did not assume a Leontief production technology. The last feature of the model complicates the analysis slightly. With a Leontief production technology, the derivative $\frac{\partial mc_j}{\partial \psi_i}$ is constant, and it will be positive given the assumption that higher quality inputs demand higher prices. However, with more general production technologies, this derivative will itself depend on quality. We therefore need to show explicitly that ψ_i is an increasing function of v_j. The latter can be established using the second order conditions associated with profit maximization:

$$\begin{aligned} \frac{1}{\alpha} \cdot \kappa_i \cdot \frac{\partial v_j}{\partial \psi_i} \cdot \frac{1}{\psi_i} - \frac{1}{\alpha} \cdot \kappa_i \cdot v_j \cdot \frac{1}{(\psi_i)^2} - \frac{\partial^2 mc_j}{\partial \psi_i^2} &< 0 \\ \frac{1}{\alpha} \cdot \kappa_i^2 \cdot \frac{v_j}{(\psi_i)^2} - \frac{1}{\alpha} \cdot \kappa_i \cdot \frac{v_j}{(\psi_i)^2} - \frac{\partial^2 mc_j}{\partial \psi_i^2} &< 0 \end{aligned} \tag{A.14}$$

Let us define function $F \equiv \psi_i \cdot \frac{\partial mc_j}{\partial \psi_i} - \frac{1}{\alpha} \cdot \kappa_i \cdot v_j$. From the implicit function theorem, $\frac{\partial \psi_i}{\partial v_j} = -\frac{F_j}{F_i}$ where

$$F_j = -\frac{1}{\alpha} \cdot \kappa_i < 0 \tag{A.15}$$

and by virtue of the second order condition,

$$F_i = \frac{\partial mc_j}{\partial \psi_i} + \psi_i \cdot \frac{\partial^2 mc_j}{\partial \psi_i^2} - \frac{1}{\alpha} \cdot \kappa_i^2 \cdot \frac{v_j}{\psi_i} = \frac{1}{\alpha} \cdot \kappa_i \cdot v_j \cdot \frac{1}{\psi_i} + \psi_i \cdot \frac{\partial^2 mc_j}{\partial \psi_i^2} - \frac{1}{\alpha} \cdot \kappa_i^2 \cdot \frac{v_j}{\psi_i} > 0 \tag{A.16}$$

It follows that $\frac{\partial \psi_i}{\partial v_j} = -\frac{F_j}{F_i} > 0$. That is, input quality is an increasing function of output quality for every input.

Given the assumption that higher input quality demands a higher input price, it immediately follows that input prices will also be an increasing function of output quality for all inputs. From equation (A.2):

$$W_i(\psi_i) = \overline{W}_i + z_i \cdot \psi_i = \overline{W}_i + z_i \cdot \frac{1}{\alpha} \cdot \kappa_i \cdot v_j \cdot \left[1/\frac{\partial mc_j}{\partial \psi_i}\right]$$

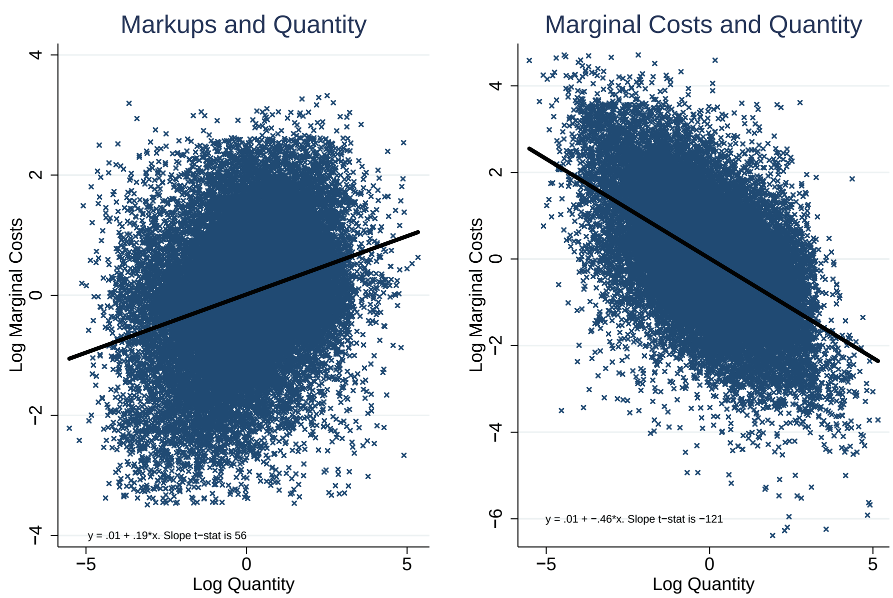
In light of the above discussion, each input price facing a particular firm can be expressed as a function of the firm's output quality, $W_i = g(v_j)$. Moreover, given that output quality is a function of output price, market share and product dummies, we have: $W_i = w(p_j, ms_j, I)$.

Appendix References

1. Berry, Steven (1994). "Estimating Discrete-Choice Models of Product Differentiation", *The Rand Journal of Economics 25(2)*, 242-262.

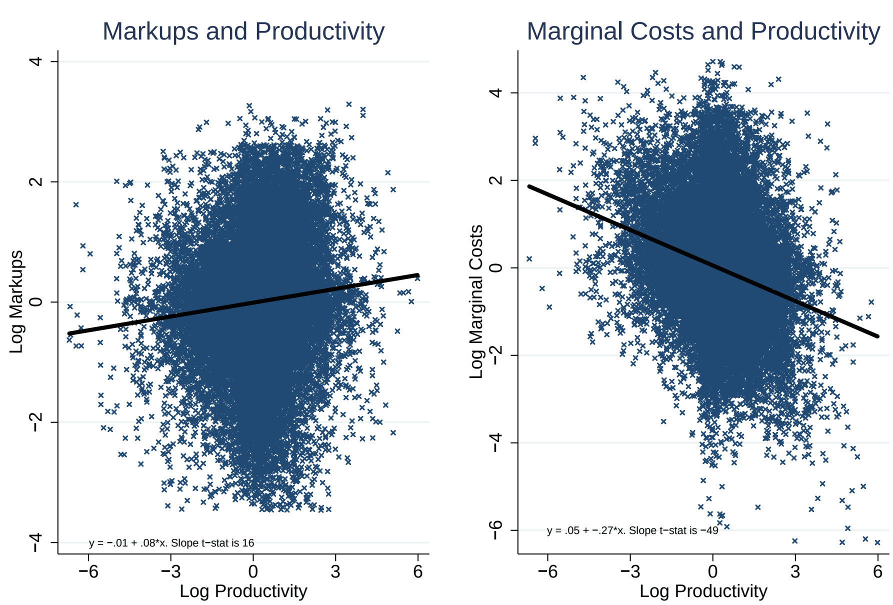
Appendix Figures

Figure A.1: Marginal Costs and Quantities



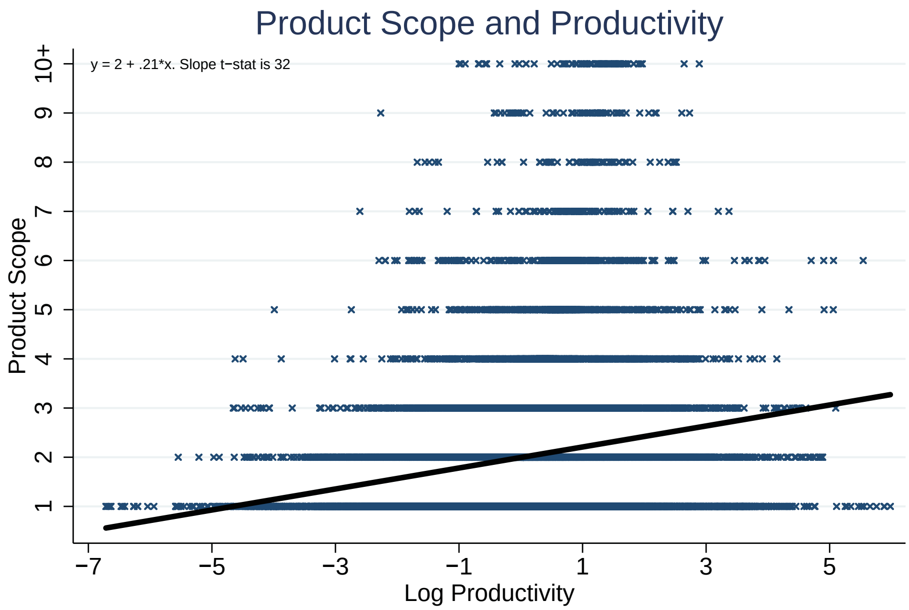
Variables demeaned by product−year FEs.
Markups, cost and quantity outliers are trimmed below and above 3rd and 97th percentiles.

Figure A.2: Marginal Costs and Productivity



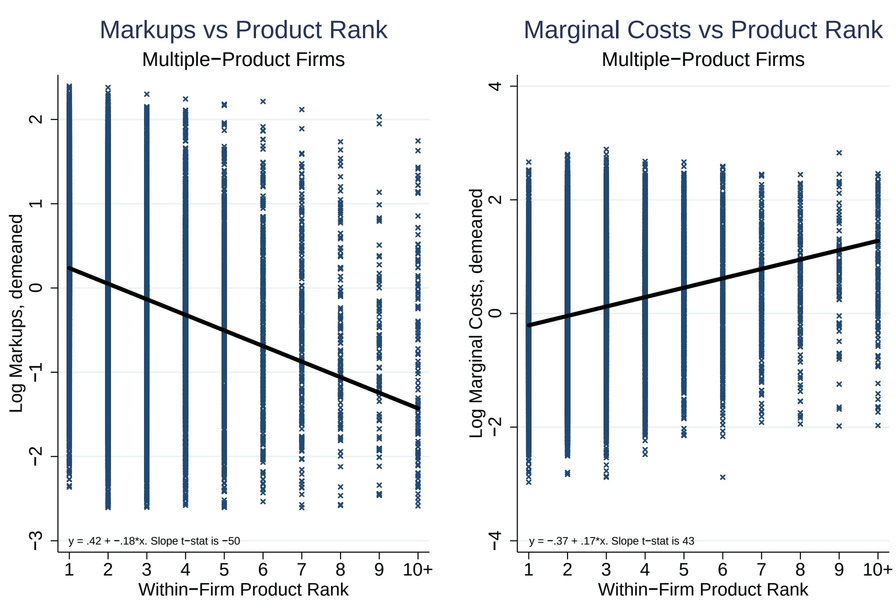
Markup and Marginal costs are demeaned by product−year FEs. Firm productivity is demeaned by the firm's main industry−year FE.
For each variable, outliers are trimmed below and above 3rd and 97th percentiles.

Figure A.3: Product Scope and Productivity



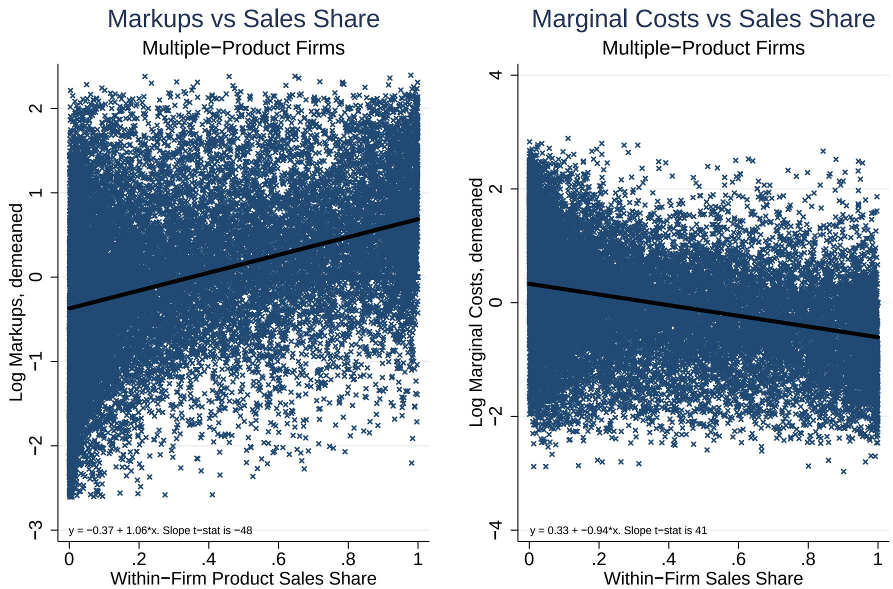
Firm productivity is demeaned by the firm's main industry−year FE.
Productivity outliers are trimmed below and above 3rd and 97th percentiles.

Figure A.4: Markups, Costs and Product Rank



Markups and marginal costs are demeaned by product−year and firm−year FEs.
Markup and marginal cost outliers are trimmed below and above 3rd and 97th percentiles.

Figure A.5: Markups, Costs and Product Sales Share



Markups and marginal costs are demeaned by product−year and firm−year FEs.
Markup and marginal cost outliers are trimmed below and above 3rd and 97th percentiles.