

Optimal Fiscal and Monetary Policy with Distorting Taxes

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# OPTIMAL FISCAL AND MONETARY POLICY WITH DISTORTING TAXES

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ABSTRACT. When the interest rate on government debt is low enough, it becomes possible to roll it over indefinitely, never taxing to retire it, without producing a growing debt to GDP ratio. This has been called a situation with zero “fiscal cost” to debt. But when low interest on debt arises from its providing liquidity services, zero fiscal cost is equivalent to finance through seigniorage. Some finance through seigniorage is generally optimal, however, despite results in the literature seeming to show that this is not so.

In recent articles Blanchard (2019) and Mehrotra and Sergeyev (2019) have reminded us that real rates of return on government debt have in most years been so low that the debt-to-GDP ratio would decline or remain stable even if the debt were simply rolled over. That is, without any taxation to “back” the debt, with interest and principal payments being financed entirely by the issue of new debt, the debt-to-GDP ratio would not increase. They characterize this situation as one in which there is zero or negative “fiscal cost” to public debt.

Conventional thinking about public debt sees debt as requiring fiscal backing, so that increased debt requires increased future taxes. If the taxes are distorting, this is a burden. But if debt has zero or negative “fiscal cost”, this conventional argument seems to fail. Indeed, it seems to lead to the conclusion that unless we expand the public debt, we are imposing an unnecessary welfare-reducing fiscal burden. Of course issuing more public debt might eventually force up interest rates, but until the “fiscal cost” is driven to zero, additional debt is providing a service people are apparently willing to pay for. Optimal fiscal policy then should, apparently, expand the debt until fiscal costs are driven to zero.

There is something to this argument, but it is essentially the same as the argument Milton Friedman made for an “optimal quantity of money”, which he said would be reached when the nominal interest rate was driven to zero. This is sometimes called the “Friedman rule”. In this paper we lay out the connection of the “fiscal cost” arguments to the Friedman rule and also explain limitations to the results in some papers in the literature that have claimed that the Friedman rule is optimal even when only distorting taxes are available.

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Seigniorage is a source of revenue and inflation is a kind of tax. If an array of distorting taxes is available, generally making some use of each of them is optimal, as lower tax rates are less distorting. It seems then likely that when only distorting taxes are available, it is optimal to make at least some use of the inflation tax, so that other tax rates can be lower.

In a classic article Chari and Kehoe (1999)<sup>1</sup> proved that in an apparently quite general setting, it is optimal not to use the inflation tax, even when the only other taxes available are distorting. That is, when money pays no interest, the nominal interest rate on other assets should be zero, or if money pays interest, the rate on it should match that on other assets.

The Chari/Kehoe result depends on an assumption, which they make explicit but do not emphasize, that the government can at an initial date inject money into the economy by purchasing bonds from the private sector. Under the commonly invoked assumption that government interest-bearing debt must remain non-negative, the Chari/Kehoe result does not hold.

To see this point, we first set out a simple model in which it is easy to prove analytically that using the inflation tax is optimal. This model satisfies the conditions that Chari and Kehoe emphasize as important for their result. As we will discuss, it does not satisfy the assumptions about the economy's initial conditions that Chari and Kehoe impose.

## I. SIMPLE MODEL

A representative individual chooses the time paths of  $C$  (consumption),  $L$  (labor),  $B$  (nominal government debt) and  $M$  (non-interest-bearing money) while taking  $P$  (the price level) and  $\tau$  (the tax rate on labor) paths as given (and known in advance). The individual's objective is to maximize

$$\int_0^{\infty} e^{-\beta t} (\log C_t - L_t) dt \quad (1)$$

subject to

$$C \cdot (1 + \gamma v) + \frac{\dot{B} + \dot{M}}{P} = (1 - \tau)L + \frac{rB}{P} \quad (2)$$

$$v = \frac{PC}{M}. \quad (3)$$

Here  $v$  is a version of "velocity", and  $\gamma v / (1 + \gamma v)$  can be interpreted as the fraction of consumption spending that is absorbed by transactions costs. An equivalent

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<sup>1</sup>Chari, Christiano, and Kehoe (1996) showed this result earlier, in a paper that assumed cash-in-advance transactions technology. The later paper by Chari and Kehoe extended the result to other transactions technologies, including that in this paper.

specification would define

$$C^* = C \cdot (1 + \gamma v)$$

and then make utility depend on

$$\log \left( \left( \sqrt{1 + \frac{4\gamma C^*}{m}} - 1 \right) \cdot \frac{m}{2\gamma} \right),$$

where  $m = M/P$  is real balances, rather than on  $\log C$ . The money-in-budget-constraint formulation makes it easier to see what fraction of spending is absorbed by transactions costs, and in particular avoids allowing implied transactions costs becoming negative. The money-in-utility version makes utility homothetic in  $m$  and  $C^*$ , as Chari and Kehoe assume.

The linear disutility of labor and unbounded supply of labor are chosen to make the algebra of deriving our results simpler. As should become clear, the results don't depend on these details of the specification.

The private agent's problem leads to first order conditions

$$C \cdot (1 + 2\gamma v) = 1 - \tau \quad (4)$$

$$\frac{-\dot{\tau}}{1 - \tau} = \gamma v^2 - \beta - \frac{\dot{P}}{P}. \quad (5)$$

$$r = \gamma v^2 \quad (6)$$

The government has to finance a given path  $G$  of expenditures, which provide no utility to the private agents. Its budget constraint is

$$\frac{\dot{B} + \dot{M}}{P} + \tau L = G + \frac{rB}{P}. \quad (7)$$

The social resource constraint

$$C \cdot (1 + \gamma v) + G = L \quad (8)$$

is derivable from the government and private budget constraints.

Suppose policy fixes constant values for  $r$  and  $\tau$ , from  $t = 0$  onwards. From (6) we see that this also fixes a constant value for  $v$ . Then the private labor-consumption tradeoff equation (4) and the social resource constraint (8) provide two equations in the two unknowns  $C$  and  $L$ . If the policy is feasible, therefore,  $C$  and  $L$  will be constant. The private sector's Euler equation (5) then becomes  $r = \beta + \dot{P}/P$  and implies that inflation is constant. Since real balances  $m = M/P$  are just  $C/v$ , they also are constant.

Substituting these relations into the government budget constraint (7) allows us to arrive at

$$\dot{b} = G - \tau L - m \frac{\dot{P}}{P} + \beta b, \quad (9)$$

where  $b = B/M$  is real debt. Everything on the right of (9) except  $b$ , we already know to be constant. With  $\beta > 0$ , this is an unstable differential equation in which all solutions but one grow in absolute value at the rate  $\beta$ . Assuming that both  $B$  and  $M$  must be non-negative, the private sector's transversality condition is

$$e^{-\beta t} \frac{b_t + m}{C \cdot (1 + 2\gamma v)} \xrightarrow{t \rightarrow \infty} 0, \quad (10)$$

where the time subscripts have been omitted on variables we know must be constant. Obviously (10) implies  $b$  cannot explode at the rate  $\beta$ , so there is only one solution to (9) consistent with private sector optimization, that with  $b$  constant at

$$b = \frac{\tau L + m \frac{\dot{P}}{P} - G}{\beta}. \quad (11)$$

That is, real interest-bearing debt is the excess of tax revenues and seigniorage over government expenditure, discounted to the present at the rate  $\beta$ . Because the government budget constraint implies a continuous time path for  $B + M$ , and  $b + m$  is determined by the equilibrium conditions, the initial price level is uniquely determined.

If (11) delivers a negative value for  $b$ , the  $\tau, r$  policy choice is not feasible — there is not enough tax revenue and seigniorage to cover  $G$ . Assuming no constraints on the ability of the policy authority to impose a surprise jump inflation or deflation at time 0, it is optimal to choose an  $r, \tau$  pair that makes  $b = 0$ . With  $b > 0$ , fiscal effort must be greater, either through seigniorage or taxation, and this is welfare-reducing.

There is a menu of choices of  $r, \tau$  pairs that deliver  $b = 0$ , and welfare differs along this menu. It is clear from (11), though, that setting  $r \doteq 0$  and thus  $m \doteq \infty$  is not feasible, much less optimal, since this would require the seigniorage term  $m\dot{P}/P \doteq -\infty$  and  $\tau L$  can be shown to be bounded above.

Where does this model deviate from the assumptions that allow Chari and Kehoe to derive their result? They observe that this model, in which the policy-maker is assumed to fully commit to future policies, has the property that it is optimal for the government to default on any outstanding net debt at the initial date, while issuing new liabilities on which it promises believably that it will never default. A policy recommendation like this is obviously unrealistic, so in models like this it is usual to constrain what the government can do at the initial date. Chari and Kehoe assume the government has zero net worth at  $t = 0$ , as would be true if it initially defaulted on outstanding debt and initially there was no money. They assume that to get money into the hands of the public at  $t = 0$ , the government undertakes “open market operations”, buying from the private sector non-money nominal assets (bonds, or loans to the public from the government). In other words, they assume that government interest-bearing debt can be negative, and arbitrarily large

in absolute value. At the initial date, their assumptions imply government has exactly matched nominal assets and nominal liabilities, with  $M$  being the liabilities,  $B = -M$ .

By promising to shrink  $M$  (and  $B$ ) at the rate  $-\beta$ , the government can then guarantee a nominal interest rate of zero. It can shrink the stock of money entirely by open market operations, selling its loans or bonds back to the public at the rate  $-\beta$ . This keeps assets and liabilities perfectly aligned, and no taxes are ever required to maintain the  $M$  shrinkage rate.

## II. WHAT ARE REASONABLE ASSUMPTIONS ABOUT INITIAL CONDITIONS?

The government budget constraint (7) implies that total government liabilities,  $B + M$  has a continuous time path. In other words, to change  $B + M$  requires changing tax revenues  $\tau L$ , government expenditures  $G$ , or interest payments on debt  $rB$ . It is unreasonable to think of any of these as being changeable at infinite rates. Treating the rate of growth  $\dot{B} + \dot{M}$  as well defined at every date, which is implied by treating (7) as a non-forward-looking equation, makes economic sense.

The government budget constraint does not require that  $B$  and  $M$  individually have continuous time paths. With  $B_{-0}$  and  $B_{+0}$  (for example) as notation for the left and right limits of  $B$  at time zero, we have

$$B_{-0} + M_{-0} = B_{+0} + M_{+0}, \quad (12)$$

while  $B_{-0} \neq B_{+0}$  and  $M_{-0} \neq M_{+0}$  are possible. But then if  $B_{-0} + M_{-0}$  is non-zero, to arrive at zero net worth at time zero requires a policy that makes  $P_{+0}$  infinite. If  $B$  is bounded below, this implies in turn real balances  $m_{+0} = M_{+0}/P_{+0}$  are zero and that transactions costs absorb all of output.

While setting interest bearing debt to zero (or its minimum feasible value) is beneficial, in the model money balances are important. An optimizing government, free to choose a time path of policy that fixes the price level at time zero, will choose a combination of interest rate and tax policies that make  $M_{+0} = B_{-0} + M_{-0}$  and thus  $B_{+0} = 0$ .

Another way to explain our initial date assumption is that we assume the government issuing nominal debt is recognized to have the option at any date of diluting the claims of holders of the debt and of money balances by deficit finance, but is assumed to be committed not to introduce a new currency, and debt denominated in it, that has a higher seniority as a claim on future revenues.

Of course if the government at time zero can repudiate both existing debt and existing currency, then issue new currency and use it to buy private sector liabilities, and if it can do all this while preserving its perfect credibility, it can achieve a better outcome than is obtainable by just setting the  $P_{+0}$  so that agents choose  $B_{+0} = 0$ . However defaulting on outstanding debt and currency is an action that the government is assumed never to do in the future. On the other hand using unanticipated

price level fluctuations to stabilize the real value  $b + m$ , and maintaining  $b \doteq 0$ , is not only optimal at time zero, in a stochastic version of the model the fully committed government does this at every date.

### III. NUMERICAL EXAMPLE

For any constant tax rate  $\tau$ , and imposing the condition that  $b \equiv 0$ , we can solve analytically (4), (5), (8) and (9) for  $C, L, v, \dot{P}/P$ , and then (via (6))  $r$ . It is therefore easy to find the optimal constant tax rate and the corresponding optimal  $r$  and inflation tax. Since the price level is allowed to jump at the initial date, this steady state is in fact the complete solution to the model for a fixed tax rate — there is no transition path to the steady state; the economy reaches the steady state immediately.

$\gamma$	$C$	$v$	$L$	$\dot{P}/P$	$U$	$\tau$	$\sigma$	$\gamma v / (1 + \gamma v)$
0.001	0.69	2.98	0.993	-0.0111	-1.363	0.305	-0.0026	0.003
0.01	0.67	0.98	0.980	-0.0105	-1.375	0.314	-0.0072	0.010
0.1	0.62	0.34	0.944	-0.0086	-1.417	0.335	-0.0160	0.033
1	0.51	0.13	0.874	-0.0040	-1.548	0.361	-0.0159	0.112

TABLE 1. Optimal steady state with  $G = .3, \beta = .02$

$\gamma$ : transactions cost parameter;  $C$ : consumption;  $v$ : velocity  $PC/M$ ;  $L$ : labor;  $\dot{P}/P$ : inflation rate;  $U$ : utility;  $\tau$ : labor tax rate;  $\sigma$ : seigniorage revenue;  $\gamma v / (1 + \gamma v)$ : proportion of consumption expenditure absorbed by transaction costs;  $G$  non-productive government expenditure;  $\beta$ : discount rate.

$\gamma$	$C$	$v$	$L$	$\dot{P}/P$	$U$	$\tau$	$\sigma$	$\gamma v / (1 + \gamma v)$
0.001	0.97	0.74	0.973	-0.0194	-1.001	0.026	-0.0254	0.001
0.01	0.94	0.35	0.943	-0.0188	-1.005	0.054	-0.0507	0.003
0.1	0.87	0.16	0.882	-0.0173	-1.024	0.103	-0.0912	0.016
1	0.72	0.08	0.778	-0.0139	-1.104	0.166	-0.1293	0.072

TABLE 2. Optimal steady state with  $G = 0, \beta = .02$

See notes to Table 1

Tables 1, 2 and 3 show the model's optimal steady state at various levels of government spending  $G$  and transactions cost parameter  $\gamma$ . Even in Table 2, where there is no government spending to finance, it is optimal to have a positive rate of labor tax  $\tau$ , in order to induce deflation and hence reduce the incentive to conserve on real balances. However even in this case inflation does not go to  $-.02$ , as would be required to make the nominal interest rate zero and match the Friedman rule. The most realistic of these cases, if we think of  $M$  as including the entire public debt in

$\gamma$	$C$	$v$	$L$	$\dot{P}/P$	$U$	$\tau$	$\sigma$	$\gamma v/(1 + \gamma v)$
0.001	0.20	9.07	0.998	0.0623	-2.627	0.800	0.0013	0.009
0.01	0.19	2.92	0.994	0.0655	-2.665	0.801	0.0042	0.028
0.1	0.17	0.98	0.983	0.0755	-2.775	0.801	0.0129	0.089
1	0.12	0.34	0.965	0.0937	-3.060	0.794	0.0342	0.252

TABLE 3. Optimal steady state with  $G = .8$ ,  $\beta = .02$ 

See notes to Table 1

the US, might be Table 1's  $\gamma = .01$  row. This produces a "debt to GDP" ratio of about 1.0 and implies transactions costs absorb about 1% of consumption and a tax rate of 31%. In this case the optimal interest rate is about one percent.

If government expenditure becomes large enough, it becomes optimal to make seigniorage positive, as we see in Table 3. In that table taxes and inflation substantially depress consumption in order to accommodate the high level of  $G$ . Inflation is over 6%, implying interest rates of over 8%. Nonetheless seigniorage delivers only a small fraction of total revenue.

With  $\tau = 0$ , seigniorage is increasing in  $v$  over the whole  $(0, \infty)$  range, and approaches .5 from below as  $v \rightarrow \infty$ . Even in Table 3, where government expenditure is taking up around 80% of output and the tax rate is about 80%, seigniorage remains well below its upper bound. Even though the Friedman rule is not optimal in this model, the inflation tax produces little revenue relative to the distortion it induces, so that it is optimal to use it to generate revenue only when the other tax available is at highly distortionary levels.

#### IV. INTEREST BEARING DEBT AND GROWTH

This model is essentially unchanged if we assume that  $M$ , instead of paying a zero nominal interest rate, pays some fixed rate  $r_m > 0$ . What matters to the equilibrium is the gap between the nominal interest rate  $\gamma v^2$  and the rate of return on  $M$ . With the interest rate on  $M$  fixed at zero, this gap is just  $\beta$  plus the inflation rate. With the rate on  $M$  fixed at  $r_m$ , the gap is  $\gamma v^2 - r_m = \beta + \dot{P}/P - r_m$ . Thus an equilibrium with inflation at  $\pi$  and no interest on money matches one with inflation at  $\pi + r_m$  when  $r_m > 0$ . In our tables above, with  $r_m = .02$ , for example, all the optimal inflation rates would be positive. Only the  $\dot{P}/P$  columns of the tables would change with  $r_m$ .

The model could also easily be modified to allow for growth, with exponentially growing labor quality (i.e. purely labor-augmenting technological change). This would change the requirement, for the Friedman rule, of deflation at the rate  $\beta$  in the  $r_m = 0$  version of the model to deflation at the rate  $\beta + v$ , where  $v$  is the rate of labor quality improvement and  $\beta + v$  is the equilibrium real rate of interest. However the qualitative conclusions from the model would remain unchanged.

## V. IMPLICATIONS FOR POLICY

Krishnamurthy and Vissing-Jorgensen (2012) present evidence that US Treasury securities have historically paid a lower yield than comparable corporate bonds, that this cannot be explained by default risk, and that the yield spread shrinks as the supply of treasury securities increases. They argue that Treasuries behave much like non-interest-bearing high-powered money in these respects. Our simple model can be regarded as a stylized equilibrium model built on their observations.

In this model with no growth in population or technology, debt has zero or negative “fiscal cost” in the sense of Mehrotra and Sergeyev or Blanchard when seigniorage is greater than or equal to zero. This is true in steady state when  $r_m - \dot{P}/P < 0$ , i.e. when the real rate of return on government debt is non-positive. When  $r_m = 0$ , the standard “money” case, this is equivalent to inflation being non-negative.

Under what conditions in this model is it welfare-improving to increase reliance on unbacked deficit finance? Since the model implies a tradeoff of one kind of finance (seigniorage) against another (labor taxation), both of which are distorting, the answer to this question must depend on both the rate of labor taxation and the “fiscal cost” (i.e. seigniorage). Increased unbacked deficits will generate revenue and allow reduction of labor taxes, or increased  $G$  with no increase in labor taxes. But this is not always welfare-improving. In our simple, casually calibrated, model, in the  $G = .3, \gamma = .01$  case, it is optimal to expand seigniorage to reduce labor taxes only when seigniorage is excessively negative. With seigniorage positive, which is equivalent to “fiscal cost” being negative, welfare can be improved by reducing inflation and seigniorage, while increasing labor taxes.

Note, though, that reducing reliance on seigniorage finance does not reduce real debt — it increases real debt. With non-interest-bearing money, moving from a steady state with higher reliance on seigniorage to one with lower reliance on seigniorage requires reducing the inflation rate, i.e. monetary contraction. With interest-bearing debt, reducing reliance on seigniorage can be achieved by expanding nominal debt while appropriately increasing the interest rate on debt. This allows expansion of real debt without deflation or reduced inflation during the transition. Of course in either case, the reduced reliance on seigniorage requires increased use of other distorting taxes.

## VI. REAL WORLD COMPLICATIONS

This paper’s model implies that high levels of real debt, at least in steady state, go with either high levels of taxation or high levels of seigniorage. After the 2008-9 crisis, most rich countries have greatly expanded their levels of real debt, while inflation has remained low and taxation has not greatly increased. Are these observations consistent with the framework of the model?

If we thought of the expansion of debt as a sudden, arbitrary policy change, the observations do not fit the model. But the policy changes clearly were responsive to developments in the economies where they took place. The financial crisis increased demand for liquid assets, both through increased private sector awareness of the risks of market disruptions and through regulatory measures that increased the need for demonstrable liquidity on balance sheets. Such a shift might be approximated by a rise in  $\gamma$  in the model. As can be seen from Tables 1-3, such a shift would, with optimal policy, lead to a steady state with lower velocity and hence higher real debt, but with modest change in the tax rate or the rate of inflation.

Under this interpretation of the expansion of the ratio of debt to GDP, the expansion is not in itself a reason for greatly increased fiscal stringency. Just as before the increase in real debt, the desirability of unbacked debt finance, i.e. of increased reliance on seigniorage, depends on comparing the distortion from constricting the supply of liquidity services from government debt to the reduced distortion from direct taxation, or the benefits from productive government expenditures, that would be allowed by increasing reliance on seigniorage. So long as inflation, interest rates on government debt, and tax rates remain as they were before the debt expansion, the optimal balance of seigniorage vs. direct taxation is not much affected by the level of debt.

Blanchard (2019), based on his Figure 15, argues that there is some evidence for a decline in the real return on capital, measured as the rate of profit relative to market capitalization. If there is such a decline<sup>2</sup>, we could interpret it in this paper's model as a decline in  $\nu$ , the rate of labor-augmenting technological improvement. This would imply that the  $r - r_m$  gap might have been declining despite the stable rate of inflation, and therefore that the economy has been moving toward less reliance on seigniorage finance. This does not in itself, of course, imply that returning to previous levels of seigniorage would be optimal.

This paper uses a representative agent model, and therefore cannot consider intergenerational tax-shifting or "crowding out" issues. Blanchard and Mehrotra/Sergeyev instead assume away tax distortions in order to focus on intergenerational issues. Both aspects of debt finance are important, and should be considered jointly. Furthermore, liquidity premia on government debt vary somewhat across the term structure and across time, and currency, bearing no interest, does exist. A more serious quantitative evaluation of the effects of debt finance should consider all of these potentially important factors as operating jointly.

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<sup>2</sup>The rate was in the 6-8% range in the 1960's in the US, and has been in the same range since 2001. There is some apparent downward trend, but it is not strong or uniform.

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